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KIRKLAND & SCOTT.

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ELEMENTARY ARITHMETIC

ON THE

UNITARY SYSTEM.

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PREFACE.

THE importance of Arithmetic as a branch of instruction is universally admitted; but, until a comparatively recent period, the results of teaching it were very unsatisfactory, and not at all commensurate with the time usually devoted to it in our schools. This was not owing to any inherent difficulty in the subject itself, but to the method of teaching it. The rule was stated first, an example illustrating the rule followed, and the reason of it came last. Now exactly the reverse of this is adopted by all good teachers. The examples and illustrations precede and lead up to the enunciation of the rule, whenever a rule is considered necessary. But while the method of teaching Arithmetic has undergone a complete change, no corresponding change has taken place in our elementary text-books. To remedy this defect the following pages have been written.

We would call attention to the general features of the work :—

1. THE UNITARY SYSTEM.—In all our best schools this system has already superseded the cumbrous and illogical methods of our ordinary text-books. Its advantages are so great that it must soon become universal. It has been defined as a method of solving arithmetical problems independently of rules by reasoning out each step of the solution from some previous one, until, by a series of deductions, the result sought is obtained. This system trains the pupils to habits of neatness, exactness, and to logical habits of thought; but its chief advantage is its extreme simplicity, dispensing with set rules, and enabling the pupil to solve problems in Simple and Compound Proportion, Simple and Compound Interest, Percentages, Profit and Loss, Partnership, etc., by one uniform, elegant, and simple process.

2. ARRANGEMENT.—The different subjects have been arranged with reference to their importance and their simplicity; the less difficult and more practical first, and the more intricate and less important afterwards. Thus, problems in Canadian Money, Bills, etc., have been introduced immediately after Division, as being of greater importance than any other subject, within the range of the pupils' ability, at that stage of their progress.

3. ORAL EXERCISES.—Each subject has been elucidated by Oral Exercises leading up to written work. This arrangement will assist

the pupil in arriving at the reasons for the methods employed, and, to a certain extent, make him the author of his own definitions and rules.

4. RULES.—The rule is given as a convenient summary of the methods employed in the solutions of the examples which precede it. The aim has been to lead the pupil to derive his own methods of operation.

5. EXERCISES—Special care has been taken in framing and selecting the exercises for the different sections in order to obtain such as will not only evolve thought on the part of the pupil, but more especially prepare him for the business relations of life.

Toronto, May, 1878.

PREFACE TO THE NEW EDITION.

This New Edition has the following improvements :—

1. It is printed on better paper, in large and clear type, thus rendering the appearance of the pages more attractive.

2. It has been carefully and thoroughly revised. To every chapter additional exercises have been added. Illustrative diagrams have been introduced, which it is hoped will simplify the teaching of the different subjects. This is more especially the case in Fractions.

3. The part that treats of Commercial Arithmetic has been re-written, re-arranged and enlarged.

4. The short chapter on Measurement in former editions has been superseded by a new chapter on Mensuration, with diagrams indicating the method of teaching that subject.

5. There has been added a new chapter on the Metric System, with illustrative examples, exercises, and easy methods of changing Metric measurements into those at present in use.

During the seventeen years that have elapsed since the publication of the first edition of this book part of the daily duties of the authors has been to examine and illustrate the best methods of teaching Arithmetic, and the experience of those years has not enabled them to find any better methods than those recommended in the following pages.

Thanking their fellow-teachers for the cordial reception given to previous editions, this New Edition is sent forth with the earnest desire that it may meet with their approbation, that it may lighten their labor, and aid them in teaching the subject of which it treats.

Toronto, November, 1895.

CONTENTS.

CHAP. I.—SIMPLE RULES.

PAGE

SECTION I.—Definitions, Notation and Numeration 1
SECTION II.—Addition 8
SECTION III.—Subtraction 17
SECTION IV.—Multiplication 26
SECTION V.—Division 38

CHAP. II.—CANADIAN MONEY. 61

CHAP. III.—MEASURES AND MULTIPLES.

SECTION I.—Prime Numbers, Prime Factors, etc..... 75
SECTION II.—Cancellation 76
SECTION III.—Highest Common Factor 77
SECTION IV.—Least Common Multiple..... 81

CHAP. IV.—FRACTIONS.

SECTION I.—Definitions 86
SECTION II.—Reduction of Fractions..... 89
SECTION III.—Addition 96
SECTION IV.—Subtraction 98
SECTION V.—Multiplication and Division..... 102
SECTION VI.—Complex Fractions 110

CHAP. V.—DECIMALS.

SECTION I.—Definitions 117
SECTION II.—Addition 120
SECTION III.—Subtraction..... 121
SECTION IV.—Multiplication..... 122
SECTION V.—Division..... 123
SECTION VI.—Reduction of Decimals 125
SECTION VII.—Circulating Decimals 126

CHAP. VI.—COMMERCIAL ARITHMETIC.

SECTION I.—Tables and Reduction..... 131
SECTION II.—Compound Addition..... 139
SECTION III.—Compound Subtraction..... 140

SECTION IV.—Compound Multiplication 142
SECTION V.—Compound Division 143
SECTION VI.—Denominate Fractions 144
SECTION VII.—Practice 147

CHAP. VII.—THE METRIC SYSTEM OF WEIGHTS AND MEASURES. 150

CHAP. VIII.—PERCENTAGES.

A. PERCENTAGES NOT INVOLVING TIME.

SECTION I.—Percentage 156
SECTION II.—Commission and Brokerage 158
SECTION III.—Insurance 159
SECTION IV.—Taxes 160
SECTION V.—Profit and Loss 161

B. PERCENTAGES INVOLVING TIME.

SECTION I.—Simple Interest 164
SECTION II.—Compound Interest 168
SECTION III.—Present Worth and True Discount 169
SECTION IV.—Bank Discount 171

CHAP. IX.—PARTNERSHIP OR PROPORTIONAL PARTS.

SECTION I.—Sharing 173
SECTION II.—Partnership without Time 174
SECTION III.—Partnership with Time 175
SECTION IV.—Averages 176

CHAP. X.—SQUARE ROOT. 178

CHAP. XI.—MEASUREMENT OF SURFACES AND SOLIDS.

SECTION I.—The Rectangle 182
SECTION II.—Carpeting Rooms 183
SECTION III.—Papering a Room 184
SECTION IV.—The Parallelogram 185
SECTION V.—The Triangle 186
SECTION VI.—The Right-Angled Triangle 187
SECTION VII.—The Circle 188
SECTION VIII.—Rectangular Solids 189

CHAP. XII.—MISCELLANEOUS PROBLEMS. 192

ANSWERS 202

ELEMENTARY ARITHMETIC

ON THE

UNITARY SYSTEM.

CHAPTER I.

Section I.—Definitions; Notation and Numeration.

1. Arithmetic is the science of numbers and the art of computing by them.

2. A Unit is a single thing regarded as a whole; as *one,* one boy, one dollar, one cent.

In counting eggs in a nest, the unit is *an egg.*

In selling eggs by the dozen, the unit is *a dozen.*

In selling bricks by the thousand, the unit is *a thousand.*

The unit, is, therefore, the standard by which we count.

3. A Number is a unit or a collection of units, and is the answer to the question, "How many?" *one* dollar is a *unit; five* dollars is a collection of units.

4. In common arithmetic, all numbers are expressed by means of the significant figures

1 2 3 4 5 6 7 8 9

called *one, two, three, four, five, six, seven, eight, nine,* and the figure 0, which is called a *cipher* or *naught,* and which has no value in itself.

5. Numbers are considered as being either **Abstract** or **Concrete.**

A **Concrete Number** is one applied to a particular unit; as 5 *men*, 6 *horses*, 9 *dollars*.

An **Abstract Number** is one not applied to any particular unit; as 3, 6, 8.

Exercise i.

1. How many units in 5? In 5 books? In 9 pencils?

2. What is the unit of 5? Of 5 books? Of 3 balls?

3. State which are abstract and which concrete of the following numbers:

6, 7, 8 books, 9 men, 3, 4, 5 apples, 2, 1 cent.

4. What is the unit of 8 miles? 9 miles? 7? 6 cents?

5. Which numbers have the same unit in the following: 3 apples, 7 apples, 4 boys, 7, 6 apples, 9 boys, 2 cents, 4 girls, 5 cents, 9, 8, 5 girls.

NOTATION AND NUMERATION.

The Arabic System.

6. Notation is the art of writing in figures any number expressed in words.

7. Numeration is the art of reading in words any number expressed in figures.

8. All **Numbers** can be expressed in figures by means of the nine *significant figures* and *naught*, as follows:

1. All whole numbers under ten are expressed by means of the nine significant figures.

2. The value of any figure is increased ten-fold by writing a figure on the right of it

It follows that

1. **Ten** may be expressed by writing 1 and 0 on its right, thus 10; for the value of the 1 is increased ten-fold by the naught which follows it.

Similarly,

Twenty, Thirty, Forty, Fifty, Sixty, Seventy, Eighty, Ninety may be expressed thus :

20, 30, 40, 50, 60, 70, 80, 90, respectively.

2. **Numbers between ten and twenty, between twenty and thirty,** etc., may be expressed by an application of the preceding statements, thus :

Ex. *Express in figures the number seventy-eight.*

We have first to express the number **seventy,** and as *seventy is ten times seven,* we write down 7 and then make it *seventy* by writing some other figure after it. Now, we might use a cipher for this purpose, but since we have to express *eight* besides *seventy,* we write an 8 after the 7, and then read the figures as *seventy-eight.*

Note.—The principle of forming and expressing numbers should be illustrated by making bundles of wooden splints, ten in each bundle, and then making bundles of hundreds by taking for each hundred ten bundles of ten each, and so on.

Exercise ii.

Write in figures :

1. Seven ; nine ; four ; two.
2. Thirty-six ; eighty-four ; twenty ; sixty-nine.
3. Forty-four ; seventy ; ninety-six ; sixteen.
4. Fourteen ; twelve ; thirty-nine ; fifty-six.
5. Write as one number,
   four tens and eight units ; nine tens and seven units ; three tens and six units ; six tens.

Write in words the numbers expressed by the following figures :

6.	7,	11,	15,	19,	59,	84,	96,	98.
7.	71,	12,	28,	91,	44,	17,	22,	34.
8.	20,	37,	48,	76,	99,	69,	70,	87.
9.	14,	35,	89,	78,	54,	49,	50,	13.
10.	90,	80,	39,	28,	11,	19,	27,	31.

9. A **Hundred** may be expressed by writing *ten*, 10, and then placing a 0 after it thus, 100: for the value of the number 10 is increased ten-fold by writing a figure after it.

Similarly,

Numbers between one hundred and two hundred, between two hundred and three hundred, etc., may be expressed by an application of the statements in Art. 8 thus:

Ex. *Let it be required to write Eight Hundred and Seventy-eight.*

Since **Eight Hundred** may be expressed by 800, and **Seventy-eight** by 78, we are able to express **Eight Hundred and seventy-eight** by 878, *i.e.*, by substituting the figures 7, 8, in place of the two ciphers in 800.

Exercise iii.

Write in figures the following numbers:

1. One hundred and forty-nine: three hundred and eight; nine hundred and seventy-four.

2. Two hundred; four hundred and twenty; six hundred and ninety-four.

3. Five hundred and sixty: nine hundred and eight: four hundred and forty-four.

4. 7 hundreds, 3 tens and 5 units; 9 hundreds and 6 tens; four hundreds and 6 units.

5. 3 hundreds and 9 units; 8 tens, 6 hundreds and 7 units; 2 units, 7 tens and 4 hundreds.

Write in words the numbers expressed by the following figures:

6.	207,	371,	185,	190,	368.
7.	570,	472,	807,	909,	990.
8.	368,	584,	760,	321,	999.
9.	394,	786,	475,	782,	700.
10.	506,	300,	407,	740,	397.

10. Numbers which consist of more than three figures are divided into *periods* or groups of *three* figures, counting always from the *right* hand side.

11. The names of the periods commencing at the right are *Units*, *Thousands*, *Millions*, *Billions*, *Trillions*, etc.

12. The places in any period have the same name as in the *units* periods, and each place must be filled with a cipher, if not occupied by a significant figure.

Ex. 1. *Write in figures seventy-eight thousand and sixty-four.*

Thousands	Units.
78	064

Ex. 2. *Write in figures twenty millions, six hundred thousand and seven.*

Millions	Thousands	Units.
20	600	007

Express in figures the following numbers:

1. Six thousand and six; four thousand three hundred; nine thousand and eighty.

2. Three thousand seven hundred; seven thousand nine hundred and six; three thousand and eighty-four.

3. Sixty-four thousand and nine; eight hundred and seven thousand and sixty-eight; seven hundred thousand three hundred and sixteen.

4. Four millions, thirty thousand and ninety-seven; eight hundred and nine millions, seven thousand and thirty-nine; five hundred and eighty-six millions and seven.

5. Eight billions; sixty-four billions, seven millions and twenty-four; four billions, four millions and four.

6. Four hundred and eight millions, three thousand and nine; seventy-four billions, seventy-four thousand and four; five hundred billions and five hundred.

7. Eighty billions and seventy millions; eight hundred millions and eight; three hundred billions, three hundred thousand and ninety.

8. Fifty-seven billions, seven hundred millions and eighty; eleven millions and eleven; nineteen billions and fourteen thousand.

9. Seven trillions and seventy; four hundred millions and one; six hundred trillions, six hundred billions and six hundred.

10. Ninety-nine trillions and eight; seven hundred billions, seventy millions and seven thousand; sixteen trillions, sixteen billions and sixteen.

Write in words the numbers expressed by the following figures:

11. 7077, 85079, 56950, 473628.

12. 56418, 784006, 400507, 360004.

13. 300071, 901007, 720009, 182010.

14. 3140006, 50000600, 3600010070.

15. 51636207640, 70000000100, 920070070070.

THE ROMAN NOTATION.

13. The system of Notation described above is the one in general use at the present time, and is called the "ARABIC NOTATION" because it was introduced into Europe by the Arabs, who had obtained it from the Hindus. Another method was in use among the Romans but is now employed only to denote the chapters and sections of books, etc. The following is a brief description of this notation:

1st. Instead of *figures* being used to express numbers, the following *letters* are employed, viz.:

I, V, X, L, C, D, M, of which the simple values are respectively:

1, 5, 10, 50, 100, 500, 1000.

2nd. *If two characters of the same value are placed side by side, or if a character is followed by one of less*

value than itself, the number denoted by the expression is the *sum* of the simple values, thus, XX represents 20; XI denotes 11.

3rd. *If a character is followed by one of greater value than itself*, the number denoted by the expression is the *difference* of their simple values, thus, IX represents 9; XL represents 40.

To write any number in Roman Numerals. Resolve the number into its different parts and always write down one part before proceeding to another, beginning at the left hand side.

Ex. Express 1895 in Roman numerals:—

1895 = 1000, 800, 90, and 5.
1000 = M
800 = DCCC
90 = XC
5 = V;
Hence, 1895 = MDCCCXCV.

Exercise V.

Write in Roman numerals:

1. 19, 24, 49, 84, 99.
2. 187, 208, 781, 962, 999.
3. 1301, 1390, 1684, 1815, 1878

Write in figures—

4. XLIV, LXIX, XCIV, LXXI.
5. XCIX, CXXIX, CLXXVII.
6. DLV, MDCIV, MDCCCXIX, MXC.

Review Exercise.

1. Define unit and number, and distinguish between abstract and concrete numbers. Give examples.

2. Whence was the ordinary system of notation derived? What method did the Romans adopt to represent numbers?

3. Express in figures the first hundred numbers with their respective names.

4. Write the smallest and the largest number possible with the following five characters: 0, 1, 2, 3 4, and express each number in words.

5. Write the different ways in which each of the nine digits can be made up of two less numbers.

6. The number 27 is composed of 16 and 11. Write all the other two numbers which can make up the number 27.

7. How many tens, how many hundreds, how many thousands, and how many ten thousands are there in a million of units?

Section II.—Addition.

1. James had 3 marbles and John gave him 2 more; how many has James now?

2. How many are 5 apples and 4 apples?

3. How many are 2 books and 3 books and 1 books?

4. John has 8 cents, his father gives him 5 cents more: how much money has he now?

5. Jane spent 5 cents on candy, 9 cents on a slate, and then had 6 cents left; how much had she at first?

Oral Exercises

1. How many are 5 and 9? 15 and 9?
2. How many are 6 and 7? 16 and 7? 16 and 7?
3. How many are 1 and 8? 21 and 8? 81 and 8?
4. Count by 2's as far as 30.
5. Count by 4's from 3 to 51.
6. Count by 6's from 4 to 76.
7. Count by 7's from 4 to 95.
8. Add any two adjacent numbers of the following: 3, 7, 8, 2, 5, 9, 7, 6, 4, etc.

9. Tell the sums of:
7 and 6, 17 and 6, 27 and 6, 37 and 6, etc.
7 and 7, 17 and 7, 27 and 7, 37 and 7, etc.
8 and 5, 18 and 5, 28 and 5, 38 and 5, etc.
9 and 8, 19 and 8, 29 and 8, 39 and 8, etc.

10. Make a circle, put one of the nine digits in the centre and the others around the circumference. Add the one in the centre to each of the others. Do this with each of the nine digits in turn.

11. A farmer sold some oats for 7 dollars, and a ton of hay for 9 dollars; how many dollars did he receive for both?

12. Paid 8 cents for raisins, and 9 cents for cloves; how many cents did both cost?

13. There are 9 boys in one class and 7 in another; how many in both classes?

14. If you work 8 examples in arithmetic to-day and 7 to-morrow, how many will you work in both days?

15. There are 9 birds on one tree and 10 on another; how many birds on both trees?

16. A lady sold 10 pounds of butter at one time, 12 pounds at another, and 3 pounds at another; how many pounds did she sell in all?

14. Combining two or more numbers of the same kind, so as to make *one* number, is called **Addition.**

15. The number found by adding two or more numbers is called their **Sum.**

16. The numbers which are added together are called **Addends.**

17. The *sign* of Addition, +, is called *Plus*, and when placed between two numbers shows that they are to be added.

Note.—The following is a convenient mode of giving a class practice in addition: Write the nine digits on the Black Board. Point to any digit, then to another, etc., the pupils adding the digits as they are pointed to. When the sum is sufficiently large let the pupils write it on their slates. In this way one sum on the Board will serve for many examples.

18. The sign, $=$, is called the *Sign of Equality*, and when placed between two numbers shows that they are equal. Thus, $2+3=5$, and is read, 2 plus 3 equals 5.

19. Like or similar numbers are such as have the same unit, as 6 *boys*, 8 *boys*, 9 *boys*, etc.

20. Principle.—Only like numbers can be added. Thus, 4 cents and 7 cents can be added together, but not 4 cents and 7 marbles.

21. Addition may be divided into two cases:

1. *Addition of numbers in which the sum of any column is less than ten.*
2. *Addition of numbers in which the sum of any column exceeds nine units of that column.*

Case I.

22. To add any column of Figures whose sum does not exceed nine.

Ex. 1. How many are 21 cents, 15 cents, and 12 cents?

21 cents.
15 "
12 "

48 cents.

Write the numbers as above, placing *units* under *units*, and *tens* under *tens*, and begin at the right to add. Thus, 2 and 5 are 7, and 1 are 8, which we write in the place of units; adding the tens we have 1 and 1 are 2, and 2 are 4, which we write in the tens' place. Hence the entire sum is 48 cents.

After a little practice the pupil should proceed as follows: 2, 7, 8; set the 8 in the units' column. Next, 1, 2, 4; set the 4 in the tens' column.

Exercise vi.

(1)	(2)	(3)
16 horses.	18 boys.	12 girls.
21 "	20 "	14 "
10 "	60 "	13 "

(4)	(5)	(6)	(7)
421	312	241	405
132	231	134	363
425	413	523	121

(8)	(9)	(10)	(11)
342	213	143	351
406	305	322	204
131	461	232	243

(12)	(13)	(14)	(15)
240	650	513	408
401	122	106	371
357	126	260	110

(16)	(17)	(18)	(19)
2341	3213	4021	1056
3214	2340	1045	6131
3034	4326	3923	2802

(20)	(21)	(22)	(23)
23241	31042	12304	21304
31402	24535	35242	30562
44235	32411	41452	28122

(24)	(25)	(26)	(27)
123402	213456	413215	325231
341256	435230	234344	253608
333240	120303	142130	410150

Exercise vii.

Practical Problems.

1. A boy spent 23 cents for a melon, 32 cents for peaches, and 24 cents for pears; how many cents did he spend?

2. Of the trees in an orchard, 23 are peach trees, 10 are plum trees, 12 are pear trees, and 43 are apple trees; how many trees are there in the orchard?

3. A farmer has 323 acres in cotton, 121 acres in corn, 123 acres in wheat, and 101 in oats; how many acres has he in cultivation?

4. A gentleman paid 225 dollars for a buggy, 231 dollars for a horse, 300 dollars for a carriage, and 40 dollars for harness; what did he pay for all?

5. A man travelled on the cars for four days as follows: the first day he went 313 miles, the second day 242 miles, the third day 220 miles, and the fourth day 214 miles; how many miles did he travel in the four days?

6. A merchant bought four bales of cloth; the first bale measured 305 yards, the second 213 yards, the third 240 yards, the fourth 211 yards; how many yards did he buy?

7. Four merchants loaded a freight train with cotton; the first put on 213 bales, the second 232 bales, the third 312 bales, and the fourth 121 bales; how many bales were put on?

8. A merchant went to Montreal and invested 5213 dollars in dry goods, 2131 dollars in groceries, 1000 dollars in hardware, and 345 dollars in confectionery; how much did he invest?

9. Four men built a factory; the first furnished 13211 dollars, the second 31123 dollars, the third 43230 dollars, and the fourth 10122 dollars; what was the cost of the factory?

10. The population of 4 cities is as follows: the first contains 321213 persons, the second 213310 persons, the

third 145245 persons, and the fourth 220130 persons; what is the total population?

Case II.

23. To add when the sum of any column exceeds Nine Units of that column.

Ex. 2. Find the sum of 358, 369, 328, and 9.

$$\begin{array}{r} 358 \\ 369 \\ 328 \\ 9 \\ \hline 1064 \end{array}$$

For convenience in adding, write the numbers, placing units under units, tens under tens, etc. Begin at the column of the lowest order: 9, 17, 26, 34: 34 units = 3 tens and 4 units. Write the 4 under the units' column and add the 3 tens with the column of tens; thus, 5, 11, 16: 16 tens = 1 hundred and 6 tens. Write 6 under the column and add 1 with the column of hundreds; thus, 4, 7, 10: 10 hundreds = 1 thousand and 0 hundreds. Write 0 under the column of hundreds, and write 1 under the column of thousands, making the sum 1064.

This may be illustrated with a few bundles of splints bound together with India rubber bands, as follows:

$$\begin{array}{rl} 358 = & 3 \text{ hund.} + 5 \text{ tens} + 8 \text{ units.} \\ 369 = & 3 \text{ “} + 6 \text{ “} + 9 \text{ “} \\ 328 = & 3 \text{ “} + 2 \text{ “} + 8 \text{ “} \\ 9 = & \qquad\qquad 9 \text{ “} \\ \hline & 9 \text{ hund.} + 13 \text{ tens} + 34 \text{ units} \\ & = 9 \text{ hund.} + 1 \text{ hund.} + 3 \text{ tens} + 3 \text{ tens} + 4 \text{ units} \\ & = 10 \text{ hund.} + 6 \text{ tens} + 4 \text{ units} \\ & = 1064. \end{array}$$

24. *PROOF.*—Begin at the top of the units' column and add the several columns downwards; if the two results agree, the work may be presumed to be correct.

Exercise viii.

(1)	(2)	(3)	(4)
42 dollars.	16 cents.	55 boys.	48 girls.
28 "	18 "	13 "	25 "
43 "	44 "	84 "	72 "

(5)	(6)	(7)	(8)	(9)	(10)
45	84	16	46	84	95
69	72	61	64	46	50
32	91	85	51	87	68

(11)	(12)	(13)	(14)	(15)	(16)
642	272	615	465	956	925
317	447	421	641	508	575
872	638	879	818	467	259

(17)	(18)	(19)	(20)	(21)	(22)
752	312	253	897	156	851
423	426	541	111	481	318
709	151	422	313	423	805
820	737	735	625	782	167

(23)	(24)	(25)	(26)	(27)	(28)
4813	1122	2291	3574	4119	1357
5914	7914	5723	3333	2575	2468
6115	1234	2102	4680	4404	5555
7036	8024	6838	3391	3685	6666

(29)	(30)	(31)	(32)	(33)	(34)
5788	3455	2729	4044	3282	1185
2693	6521	8272	5260	6341	5073
1112	6817	3228	3788	3161	9962
6762	7773	9561	5473	2827	9467
8104	6839	5587	2667	7214	3478

(35)	(36)	(37)	(38)
43474	73422	77823	13536
38242	75638	21684	71882
67891	18208	18516	81385
84870	32378	33902	80246
22171	27225	14656	91257

(39)	(40)
433827	28513534
563725	47224456
434958	31821745
367624	18714924
233647	73584627

Find the sum—

41. Of $6472 + 8733 + 4633 + 4854$.
42. Of $2762 + 8756 + 9783 + 4578$.
43. Of $1617 + 8743 + 7284 + 9621$.
44. Of $2650 + 4062 + 8705 + 9030$.
45. Of $5005 + 6007 + 7583 + 4783$.
46. Of $27845 + 67832 + 74281 + 68432$.
47. Of $47823 + 68421 + 70070 + 60504$.
48. Of $127 + 6434 + 7805 + 66782$.
49. Of $10 + 8756 + 405 + 66782$.
50. Of $7560 + 804 + 7854 + 87400$.
51. Of $1525 + 920 + 820 + 16 + 37800$.

Exercise ix.

Practical Problems.

1. A gave 27 dollars for a cow, 45 dollars for an ox, and 150 dollars for a horse; what did they all cost?

2. A has 120 acres of land, B has 310 acres, C has 515 acres, and D has 715 acres; how many acres have they all together?

3. There are 31 days in January, 28 in February, 31 in March, and 30 in April: how many days are there in these four months?

4. A man travelled 215 miles one week, 195 the next, 273 the next, and 378 the next: how far did he travel?

5. A weighs 127 pounds, B 215 pounds, C 176 pounds, D 184 pounds, and E 231 pounds; what is the sum of their weights?

6. A farmer raised 576 bushels of corn, 918 bushels of oats, 3149 bushels of wheat, and 2785 bushels of rye: how many bushels did he raise in all?

7. A owns 214 acres of land, B owns 719 acres, C owns 2136 acres, and D owns 372 acres: how many acres do they own altogether?

8. A bought a horse for 168 dollars, and a carriage for 376 dollars, and sold them so as to gain 89 dollars: how much did he receive for them?

9. In one book there are 725 pages, in another book there are 327 pages, and in another book there are as many as in both the former: how many pages in all?

10. A merchant bought cloth for 756 dollars, silk for 859 dollars, muslin for 367 dollars, and calico for 255 dollars; how much did all cost?

11. A paid 325 dollars for a span of horses, and 248 dollars more than this for a carriage: for how much must he sell them both to gain 275 dollars?

12. A gains in one year 165 dollars, B gains 136 dollars more than A, and C gains as much as A and B together: how much did B gain? how much did C gain? how much did they all gain?

13. A man dying willed his estate as follows: To his wife, 5000 dollars; to each of his four sons, 3200 dollars; to each of his three daughters, 2800 dollars; to a church, 1950 dollars; to a school, 2430 dollars; how much was his estate?

Section III.—Subtraction.

1. John had 5 cents, and bought an orange for 2 cents; how many cents had he left?

2. Mary had 6 cups, but broke 3; how many has she remaining?

3. A man, earning 10 dollars a week, spent 6 dollars for provisions; how many dollars has he left?

4. If a merchant has 12 barrels of flour, and he sells 7 of them, how many barrels has he left?

5. If you have 27 dollars, and spend 12 dollars, how much will you have remaining?

Oral Exercises.

1. Subtract by 2's from 100 to 2: thus, 2 from 100 leaves 98, 2 from 98 leaves 96, and so on.

2. Subtract by 3's from 100 to 1; by 4's from 100 to 0.

3. Subtract by 4's from 95 to 3; by 5's from 100 to 0.

4. Subtract by 6's from 100 to 4; by 7's from 100 to 2.

5. Subtract by 7's from 99 to 1; by 8's from 100 to 4.

6. Subtract by 9's from 100 to 1; by 9's from 99 to 0.

7. Count by 4's from 3 to 39, and back again to 19.

8. Count by 5's from 6 to 66, and back again to 26.

9. Count by 7's from 18 to 53, and back again to 11.

10. Count by 8's from 25 to 65, and back again to 1.

11. Make a circle and place one of the 9 digits in the centre, and around the circumference write the numbers from 10 to 20. Subtract the number in the centre from each of the numbers on the circumference. Do this with each of the nine digits in turn.

12. Jane is 11 years old, and Mary is 7 years younger; what is Mary's age.?

13. A grocer sold tea for 10 dollars and thus gained 3 dollars; what did the tea cost him?

14. If I buy cloth for 7 dollars, at what price must I sell in order to lose 4 dollars?

15. Mary has 9 dollars; she pays 7 dollars for a dress, and then earns 3 dollars more; how much has she now?

16. A boy having 12 apples, bought 6 more, and then sold 8; how many had he left?

17. James had 5 dollars, he earned 5 dollars more, and then spent 6 dollars; how much did he then have?

18. A merchant gave 8 dollars for a certain article, and paid 4 dollars for carriage; at what price must he sell to gain 3 dollars?

25. Finding the difference between two numbers is called **Subtraction.**

26. The number found by taking one number from another is called the **Difference** or **Remainder.**

27. The number from which the other is taken is called the **Minuend.**

28. That which is taken from the Minuend is called the **Subtrahend.**

29. The sign of subtraction, $-$, is called *Minus*, and when placed between two numbers shows that the one on the right of the sign is to be taken from the one on the left of it. Thus $6 - 2$, is read 6 minus 2, and means that 2 is to be taken from 6.

30. Principle. Only similar numbers can be subtracted; thus, 4 boys from 7 boys; 6 cents from 8 cents, &c.

31. Subtraction may be divided into two cases:

1. *When no figure of the subtrahend is greater than its corresponding figure of the minuend.*
2. *When a figure of the subtrahend is greater than the corresponding figure of the minuend.*

Case I.

32. To subtract when no figure of the subtrahend is of greater value than its corresponding figure of the minuend.

Ex. 1. A grocer bought 678 oranges, and sold 335 of them; how many has he left?

678 oranges.
335 "
343 "

Here we are required to find the *difference* between 678 and 335. We write the less number under the greater, placing *units* under *units* and *tens* under *tens*. Beginning with the units we say 5 units from 8 units leave 3 units, and we set the 3 in the units' column below. Then 3 tens from 7 tens leave 4 tens, and we set the 4 in the tens' column. Lastly, 3 hundreds from 6 hundreds leave 3 hundreds, and we set the 3 in the hundreds' column. Hence we have as the whole remainder 3 hundreds 4 tens and 3 units, or 343.

Exercise X.

(1)	(2)	(3)	(4)	(5)	(6)
625	456	763	617	767	896
312	215	512	215	123	432
(7)	(8)	(9)	(10)	(11)	(12)
279	807	796	736	967	875
136	502	452	432	234	345
(13)	(14)	(15)	(16)	(17)	(18)
8763	9076	3769	5076	4872	7659
4321	4054	1546	3075	2342	3237
(19)	(20)	(21)	(22)	(23)	(24)
8769	4876	8275	8799	8591	5857
3257	2142	3251	2542	7230	1234

(25)	(26)	(27)	(28)	(29)	(30)
784	82345	57596	72578	27397	67385
361	22121	21321	41362	22315	24121

(31)	(32)	(33)	(34)	(35)
57897	67858	87578	96754	81296
21472	32721	21335	21423	20135

(36)	(37)	(38)	(39)	(40)
253786	474589	87695	56728	98785
213123	212324	23542	21306	21342

(41)	(42)	(43)	(44)	(45)
373967	873972	72587	95837	89976
212851	132421	51234	51321	32742

46. 314 from 678.
47. 425 from 658.
48. 561 from 789.
49. 254 from 576.
50. 437 from 869.

51. 1235 from 3768.
52. 3726 from 4969.
53. 2532 from 8748.
54. 4720 from 87856.
55. 12345 from 68799.

Exercise xi.

Practical Problems.

1. In a school of 74 pupils, 31 are boys, how many girls are there?

2. A girl had 75 cents and paid 31 cents for a slate: how many cents had she left?

3. A man bought a horse for 98 dollars, and sold it for 82 dollars; how much did he lose?

4. Two parties played a game of base ball and made 87 runs. One party made 53 runs; how many did the other party make?

5. Jane and Susan together answered 87 questions in geography. Jane answered 43 of them; how many did Susan answer?

6. A gentleman bought a buggy for 225 dollars, and sold it for 268 dollars; what was his profit?

7. A man bought a horse for 265 dollars, and sold it for 232 dollars; how much did he lose?

8. A man deposited 5237 dollars in the bank; he afterwards drew out 3125 dollars; how much remained?

9. A man dying, left 27894 dollars to his son and his daughter. The share of the son was 13452 dollars; what was the daughter's share?

Case II.

33. To subtract when a figure in the Subtrahend is of greater value than its corresponding figure in the Minuend.

Ex. 2. From 522 subtract 285.

522
285

237

We begin at the right, but as we cannot take 5 *units* from 2 *units*, we take 1 *ten* from the 2 *tens*, and adding the 1 ten, =10 *units*, to the 2 units, we have 12 *units*. Then 5 units from 12 units leave 7 units, which we write under the units' column. Now as we took 1 ten from the 2 tens, we left only 1 ten. As we cannot take 8 *tens* from 1 *ten*, we take 1 *hundred* from the 5 *hundreds*, and considering the 1 *hundred* taken as 10 *tens*, we add it to the 1 *ten*, making it 11 tens; then 8 tens from 11 tens leave 3 tens, which we write in the tens' column. Now, as we took 1 hundred from 5 hundreds, we left only 4 hundreds: hence we say, 2 hundreds from 4 hundreds leave 2 hundreds, which we write in the hundreds' column, making the remainder 2 hundreds 3 tens and 7 units, or 237.

This process may be illustrated by splints as follow:

$$\begin{aligned} 522 &= 5 \text{ hund.} + 2 \text{ tens} + 2 \text{ units.} \\ &= 5 \quad " \quad + 1 \quad " \quad + 12 \quad " \\ &= 4 \quad " \quad + 11 \quad " \quad + 12 \quad " \\ 285 &= 2 \quad " \quad + 8 \quad " \quad + 5 \quad " \\ \hline & \quad 2 \text{ hund.} + 3 \text{ tens} + 7 \text{ units.} \\ &= 237. \end{aligned}$$

This is usually called the "*Borrowing*" or "*Decomposition*" method.

There is another method of performing subtraction, which depends on the following principle:

The difference between two numbers remains the same when each of them is increased by the same number.

For example, $5 - 2 = 3$. Now, if we add 10 to each, we have $15 - 12 = 3$, as before.

In Ex. 2, if we add 10 units to 2 units we have 12 units. Then 5 units from 12 units leave 7 units, which we write in the units' place. Now as we added 10 units to the minuend, if we add an equal number to the subtrahend the difference will remain the same. But 10 units = 1 ten. Adding 1 ten to 8 tens we have 9 tens; and as we cannot take 9 tens from 2 tens, we add 10 tens, thereby making 12 tens; then 9 tens from 12 tens leaves 3 tens, which we write in the tens' place. Since we added 10 tens to the minuend, we must add an equal number to the subtrahend, in order that the difference may remain the same. But 10 tens = 1 hundred. Adding 1 hundred to 2 hundreds we get 3 hundreds; and taking 3 hundreds from 5 hundreds we get 2 hundreds, which we write in the hundreds' place. *This is the method usually employed. This is called the "Borrowing and Carrying" or "Equal Addition" method.*

34. *PROOF.* Add the remainder to the subtrahend; the sum will equal the minuend if the work is correct.

Exercise xii.

(1)	(2)	(3)	(4)	(5)	(6)
573	718	835	968	839	538
248	375	573	675	584	394
(7)	(8)	(9)	(10)	(11)	(12)
659	839	547	658	735	848
475	583	284	372	373	539
(13)	(14)	(15)	(16)	(17)	(18)
524	752	845	307	456	450
356	387	579	138	387	382
(19)	(20)	(21)	(22)	(23)	(24)
854	943	607	500	704	403
396	765	309	325	507	285
(25)	(26)	(27)	(28)	(29)	(30)
726	857	735	792	807	650
387	389	558	295	328	357
(31)	(32)	(33)	(34)	(35)	(36)
3876	6385	6735	4076	4070	4135
2379	3527	2547	3128	2137	1216
(37)	(38)	(39)	(40)	(41)	(42)
8672	5283	8175	2534	6735	7219
3728	2426	2836	1235	5376	1972
(43)	(44)	(45)	(46)	(47)	(48)
8522	7135	6347	8135	7345	4372
6243	1872	2563	2453	2876	2583
(49)	(50)	(51)	(52)	(53)	(54)
35672	43763	87253	73875	63527	53113
23828	24235	34365	38376	14238	28401

Exercise xiii.

Practical Problems.

1. A horse was bought for 125 dollars, and sold for 117 dollars; how much was lost by the sale?

2. A roll of carpet contained 156 yards, but 79 yards were sold from it; how much remained?

3. A house cost 5440 dollars, and was sold for 6000 dollars; how much was the gain?

4. A man died in 1878 at the age of 75 years; when was he born?

5. A town which ten years ago had a population of 3745, has now a population of 6996; what is the gain?

6. I went to a store and bought a knife for 56 cents, and gave the storekeeper a four dollar bill [400 cents] to pay for it; how much change did he give me back?

7. Two little girls picked 74 quarts of blackberries one summer; if one picked 37 quarts, how many quarts did the other pick?

8. Mr. Robinson's horse and carriage cost four hundred dollars; what did the horse cost, if the carriage cost two hundred and twenty-five dollars?

9. Suppose a man's income is 2453 dollars a year, and his expenses are 1947 dollars, how much can he save in a year?

10. Smith bought of Brown 875 acres of land for 23400 dollars. For 500 acres of the tract he paid 14379 dollars; how many acres were in the remainder of the tract? And for what sum was it purchased?

Addition and Subtraction.

Exercise xiv.

Find the result of

1. $768 + 276 - 369 + 284 - 782$.
2. $369 + 784 + 468 - 266 - 368 - 248$.
3. $1764 - 839 + 786 + 724 - 368 - 256$.
4. $136 - 769 - 284 + 968 + 268 + 372$.

5. $269 - 1846 + 368 - 274 + 2976 + 769$.
6. $769 + 785 + 368 - 784 - 369 - 249$.
7. $1869 - 2846 + 362 - 489 + 3007 + 249$.
8. $2845 + 3624 - 78695 + 784 + 93768$.
9. $7369 - 245 - 12456 + 85769 - 2572$.
10. $3004 + 2006 - 5008 - 3604 + 7200$.

Exercise xv.

Practical Problems.

1. A man owing 1369 dollars, paid at one time 264 dollars, and at another 748 dollars; how much did he still owe?

2. A man bought a farm for 6780 dollars; he spent 1875 dollars for improvements and 977 dollars for stock. He then sold the whole for 9000 dollars; did he gain or lose, and how much?

3. The sum of four numbers is 936287; the first is 23789, the second is 11892 less than the first; the third is 35416 more than the second; what is the fourth?

4. What number increased by the difference between 1458 and 2362 will make the sum of 3641, 789 and 7008?

5. A collector received 1200 dollars from four men; from the first he got 352 dollars; from the second 67 dollars more, and from the third 94 dollars less than this; how much did he receive from the fourth?

6. At an election, in which there were two candidates, the whole number of votes was 3694; the defeated candidate received 1369 votes; what was the majority?

7. A boy shot an arrow up the road 173 feet, and another down the road 234 feet; his little brother brought them to him; how far did he walk to get them?

8. John and James play marbles, John has 24 at the beginning and James 36. The first game John wins 4, the next he wins 6, the next he loses 5, the next he loses 3, the next he wins 2; how many marbles has each now?

9. Find the final remainder in subtracting 54368 as many times as possible from 476209.

10. From the difference between 576 and 7852, take the difference between 19101 and 18453.

Section IV.—Multiplication.

1. There are 5 oranges in each of three dishes: how many are there altogether? 5 and 5 and 5 are how many? Three 5's or three times 5 are how many?

2. If there are 3 berries in one cluster, how many berries are there in 5 clusters? 3 + 3 + 3 + 3 + 3 is how many?

3. There are 3 feet in one yard: how many feet are there in 2 yards? In 4 yards? In 6 yards?

4. There are 6 working days in 1 week, how many working days are there in 2 weeks? In 5 weeks? 6 + 6 + 6 + 6 + 6 are how many?

5. What will 3 hats cost at 2 dollars each?

Since 1 hat costs 2 dollars, 3 hats will cost 2 + 2 + 2 dollars or 3 times 2 dollars, or 6 dollars. Hence 3 hats will cost 6 dollars.

35. When any number is to be *added* to itself a given number of times the work may be shortened by a process called **Multiplication.**

Multiplication, therefore, appears to be a short method of performing addition. It is more than this. In addition the *number* of addends is not taken into account, but in multiplication this number is always noted and is called the *multiplier*.

36. The number resulting from the Multiplication is called the **Product.**

37. The number to be added or repeated is called the **Multiplicand.**

38. The number denoting how many times the Multiplicand is to be repeated is called the **Multiplier.**

39. The *Sign* of Multiplication is formed by two short lines crossing each other slantingly: thus, ×, and is read *times* when it precedes the Multiplicand. When it follows the Multiplicand it is read *multiplied by*. In this book the sign is placed *before* the Multiplicand.

40. Principles.—

1. The *Multiplicand* may be either an abstract or a concrete number. The *multiplier* must always be regarded as an abstract number.

2. The *Product* is always of the same kind as the Multiplicand. Thus 3×3 cents are 9 cents; 2×5 boys are 10 boys.

MULTIPLICATION TABLE.

Twice	Three times	Four times	Five times	Six times	Seven times
1 is 2	1 is 3	1 is 4	1 is 5	1 is 6	1 is 7
2 .. 4	2 .. 6	2 .. 8	2 .. 10	2 .. 12	2 .. 14
3 .. 6	3 .. 9	3 .. 12	3 .. 15	3 .. 18	3 .. 21
4 .. 8	4 .. 12	4 .. 16	4 .. 20	4 .. 24	4 .. 28
5 .. 10	5 .. 15	5 .. 20	5 .. 25	5 .. 30	5 .. 35
6 .. 12	6 .. 18	6 .. 24	6 .. 30	6 .. 36	6 .. 42
7 .. 14	7 .. 21	7 .. 28	7 .. 35	7 .. 42	7 .. 49
8 .. 16	8 .. 24	8 .. 32	8 .. 40	8 .. 48	8 .. 56
9 .. 18	9 .. 27	9 .. 36	9 .. 45	9 .. 54	9 .. 63
10 .. 20	10 .. 30	10 .. 40	10 .. 50	10 .. 60	10 .. 70
11 .. 22	11 .. 33	11 .. 44	11 .. 55	11 .. 66	11 .. 77
12 .. 24	12 .. 36	12 .. 48	12 .. 60	12 .. 72	12 .. 84

Eight times	Nine times	Ten times	Eleven times	Twelve times
1 is 8	1 is 9	1 is 10	1 is 11	1 is 12
2 .. 16	2 .. 18	2 .. 20	2 .. 22	2 .. 24
3 .. 24	3 .. 27	3 .. 30	3 .. 33	3 .. 36
4 .. 32	4 .. 36	4 .. 40	4 .. 44	4 .. 48
5 .. 40	5 .. 45	5 .. 50	5 .. 55	5 .. 60
6 .. 48	6 .. 54	6 .. 60	6 .. 66	6 .. 72
7 .. 56	7 .. 63	7 .. 70	7 .. 77	7 .. 84
8 .. 64	8 .. 72	8 .. 80	8 .. 88	8 .. 96
9 .. 72	9 .. 81	9 .. 90	9 .. 99	9 .. 108
10 .. 80	10 .. 90	10 .. 100	10 .. 110	10 .. 120
11 .. 88	11 .. 99	11 .. 110	11 .. 121	11 .. 132
12 .. 96	12 .. 108	12 .. 120	12 .. 132	12 .. 144

Oral Exercise.

1. Multiply by 2 from 1 to 12; by 3 from 1 to 6.

2. Multiply by 4 from 3 to 9; by 5 from 12 to 4.

3. Multiply by 6 from 3 to 10; by 7 from 12 to 5.

4. Multiply by 8 from 12 to 2; by 9 from 1 to 11.

5. If a man drives 6 miles per hour for 7 hours, how far does he go?

Distance gone in 1 hour = 6 miles
" " 7 hours = 7 × 6 miles
= 42 miles.

6. What will 4 pair of boots cost at 6 dollars a pair?

7. A sheep costs 7 dollars; how much will 6 sheep cost at the same rate?

8. How much will 12 tons of coal cost at 6 dollars a ton?

9. A man can earn 4 dollars a day; how much can he earn in 9 days?

10. At the rate of 8 marbles for a cent, how many can be bought for 12 cents?

11. What will 7 lead pencils cost at 7 cents each?

12. If a man cuts 3 cords of wood in a day, how many cords can he cut in 12 days?

13. If 8 men can do a piece of work in 7 days, how many days will it take one man to do it?

14. How many dollars will buy 9 tons of hay at 12 dollars a ton?

15. In an orchard there are 12 rows of trees, and 11 trees in each row; how many trees are there in the orchard?

16. If a quantity of provisions will last 8 men 7 days, how many days will the same provisions last one man?

17. If 12 bushels of apples are picked from each of 8 trees, how many bushels are picked from all?

41. Multiplication may be divided into two cases:

1. *When the Multiplier does not exceed twelve.*
2. *When the Multiplier exceeds twelve.*

Case I.

42. When the Multiplier does not exceed twelve.

Ex. 1.—How many are 4 times 87 boys?

1st Operation.	2nd Operation.
87 boys.	87 boys.
87 "	4 "
87 "	
87 "	Product, 348 boys.
Sum, 348 boys.	

In the first operation we find the result by Addition.

In the second operation, which is much shorter, we write down 87 once, and we put 4, the number of times it is to be taken, under the units' figure of the Multiplicand. We then begin at the right hand side to multiply by 4; 4 times 7 units are 28 units, or 2 tens and 8 units. We write the 8 units under the units and add the 2 tens to the product of the tens. We next take 4 times 8 tens. 4 times 8 tens are 32 tens and 2 tens make 34 tens, or 3 hundreds and 4 tens. Then we write down 4 in the tens' place and 3 in the hundreds' place.

Exercise xvi.

Multiply	(1) 7432	(2) 8432	(3) 72312	(4) 92123
By	2	2	3	4

(5)	(6)	(7)	(8)	(9)
39 boys.	47 cents.	137 cows.	186 apples.	234 girls.
5	6	7	8	9

Multiply

10. 315 by 6,
11. 480 by 7.
12. 614 by 5.
13. 7842 by 3.
14. 6843 by 7.
15. 8742 by 5.
16. 9764 by 8.
17. 8973 by 6.

18. 14068 by 5.
19. 18007 by 4.
20. 82709 by 8.
21. 21876 by 7.
22. 70095 by 9,
23. 58799 by 6.
24. 71873 by 9.
25. 6742 by 8.
26. 6040 by 9.
27. 61783 by 7.
28. 60784 by 6.
29. 85643 by 5.
30. 170504 by 6.
31. 683471 by 5.
32. 863478 by 7.
33. 785473 by 8.
34. 246353 by 9.
35. 786549 by 10,
36. 832957 by 11.
37. 987356 by 11.
38. 757638 by 12.
39. 895324 by 12.

Exercise xvii.

Practical Problems.

1. What will 4070 lemons cost at 4 cents each?

2. What will 37086 oranges cost at 5 cents each.

3. A man paid 387 dollars for a house; how much should he give for 7 such houses?

4. What will 3043 pair of boots cost at 5 dollars a pair?

5. There are 56 sheep in one flock; how many sheep are there in 6 such flocks? What is the value of each flock at 7 dollars a head?

6. A man bought 384 pounds of sugar; he sold 290 pounds; how much had he left? How much did he receive for what he sold, at 9 cents a pound? What is the remainder worth at 8 cents a pound? at 7 cents a pound?

7. A merchant sold 378 kegs of nails at 9 dollars a keg; 32 hundred weight of iron at 7 dollars a hundred weight; what did each of the articles come to? What did both come to? He paid away 1389 dollars; how much money has he left?

8. I have a book with 220 pages; there are 6 paragraphs on each page; there are 9 lines in each paragraph; there are 8 words in each line; there are, on an average, 5 letters in each word; how many paragraphs are there in the book? How many lines? How many words? How many letters?

9. A grocer sold 37 pounds of rice at 8 cents a pound; 46 pounds of sugar at 9 cents a pound; what did the rice come to? What did the sugar come to? What did both come to? How much did one cost more than the other?

10. A man bought 137 pints of chestnuts at 8 cents a pint; 246 pints of peanuts at 9 cents a pint; what did each cost? What did both cost? How much did one cost more than the other?

43. To multiply by the factors of a number.

44. The **Factors** of a number are those numbers which multiplied together will produce it. Thus, 3 and 5 are the factors of 15.

Ex. 2.—Multiply 742 by 12.

$12 = 6 \times 2$, or 4×3.

742	742	742
12	6	4
8904	4452	2968
	2	3
	8904	8904

It is thus seen that the *Multiplicand multiplied by the Multiplier, gives the same product as when multiplied by any set of factors into which the multiplier can be separated.*

Exercise xviii.

Multiply

1. 478 by 25.
2. 976 by 42.
3. 1879 by 63.
4. 1362 by 49.
5. 8936 by 54.
6. 4729 by 72.
7. 2345 by 81.
8. 3764 by 64.
9. 2978 by 45.
10. 3475 by 18.
11. 7649 by 24.
12. 9365 by 144.

13. In one mile there are 1760 yards, how many yards are there in 56 miles?

14. If sound travels 1142 feet in one second, how far will it move in one minute or 60 seconds?

15. What will 72 bushels of wheat cost at 116 cents for one bushel?

16. If 27 men can do a piece of work in 17 days, how long will it take one man to do the same work?

17. What is the cost of 24 horses at the rate of 125 dollars each?

18. If a yoke of oxen costs 135 dollars, what will 63 yoke cost?

19. If a man spends 945 dollars in a year, how much will he spend at the same rate in 21 years?

20. There are 1440 minutes in a day; how many minutes are there in 28 days?

Case II.

45. When the Multiplier exceeds Twelve.

Ex. 3. Multiply 479 by 57.

$$\begin{array}{r} 479 \\ 57 \\ \hline \end{array}$$

1st partial product 3353 = 7 times the Multiplicand.
2nd " " 23950 = 5 × 10 times Multiplicand = 50 [times Multiplicand
Entire " 27303 = 57 times the Multiplicand.

Since 57 is composed of 7 units and 5 tens or 50, 57 times the number must be equal to 7 times the number, plus 50 times the number. 7 times 479 is 3353, the *first partial* product. We get 50 times 479 by first finding 10 times 479 and then multiplying this result by 5. 10 times 479 is 4790 and 5 times 4790 is 23950, the *second partial* product. We write this under the first product so that units may come under units, tens under tens, &c., and then we add the two partial products together.

In actual practice we always omit the 0.

46. *PROOF.*—Multiply the Multiplier by the Multiplicand. If the product is the same as before, the work is likely to be correct.

Exercise xix.

Multiply

1. 744 by 635.
2. 895 by 336.
3. 972 by 243.
4. 825 by 682.
5. 973 by 745.
6. 8462 by 781.
7. 9643 by 683.
8. 8532 by 763.
9. 8984 by 133.
10. 4659 by 886.
11. 28352 by 345.
12. 41678 by 287.
13. 34073 by 435.
14. 40735 by 628.
15. 29304 by 789.
16. 90705 by 897.
17. 43445 by 678.
18. 37436 by 835.
19. 88888 by 789.
20. 23567 by 597.
21. 6484 by 6372.
22. 7856 by 3375.
23. 6748 by 6334.
24. 4878 by 3437.
25. 8547 by 7733.
26. 85474 by 2547.
27. 46887 by 3489.
28. 56184 by 5474.
29. 53064 by 4871.
30. 25473 by 4487.
31. 73519 by 4735.
32. 81897 by 3456.
33. 21346 by 31452.
34. 47309 by 45233.
35. 25737 by 63252.
36. 43629 by 28516.
37. 10786 by 31672.
38. 47396 by 73462.
39. 76448 by 54173.
40. 28354 by 31867.

47. To multiply when the Multiplicand, the Multiplier, or both, contain ciphers.

Ex. 4. Multiply 2479 by 4006.

```
  2479
  4006
 -----
 14874
9916
-------
9030874
```

4006 times 2479 equals 4000 times 2479 plus 6 times 2479. 6 times 2479 is 14874; 4000 times 2479 is 9916000. These partial products are written one under the other, as before, the 0's being omitted.

Exercise xx.

Multiply

1. 415 by 307.
2. 7004 by 902.
3. 2769 by 708.
4. 1364 by 5004.
5. 9006 by 7036.
6. 1684 by 4008.
7. 2002 by 4103.
8. 3678 by 7068.
9. 9999 by 8008.
10. 3674 by 200901.

Ex. 5. Multiply 614000 by 700.

614000
700
429800000

This result is the same as that obtained by multiplying 614 by 7, and then annexing to the right *five* naughts, which is the sum of the number of naughts to the right of both the multiplicand, 614, and the multiplier, 7.

Exercise xxi.

Find the value

1. Of 473×600.
2. Of 847×700.
3. Of 9642×6300.
4. Of 1875×6340.
5. Of 27×9000.
6. Of 6000×43.
7. Of 18000×623.
8. Of 6400×640.
9. Of 650×650.
10. Of 83600×7500.
11. Of 9230×7000.
12. Of 8000×61000.

Exercise xxii.

Practical Problems.

1. In 1 ream of paper there are 480 sheets: how many sheets are there in 947 reams?

2. If a cotton mill manufactures 637 yards of cloth in one day, how many yards will it make in 307 days?

3. At 125 dollars each what will 49 horses cost?

4. A merchant bought 29 pieces of cloth; in each piece there were 57 yards: how many yards were there in the whole?

5. If 19008 pounds of hay are required for the horses of

a cavalry regiment for one day, how many pounds will be needed for 206 days?

6. What would be the cost of constructing 309 miles of plank road, at 3975 dollars a mile?

7. How many apples will an orchard containing 208 trees produce, if the average yield is 1269 apples for each tree?

8. In 3 editions of 750 books each, how many pages are there, if each book contains 407 pages?

9. How many yards of sheeting are there in 57 bales, each bale containing 25 pieces and each piece 43 yards?

10. In a cotton mill there are 29 looms; each loom can weave 42 yards daily. At this rate how many yards can be woven in 159 days?

11. A lot cost 420 dollars; how much will 105 lots cost at the same rate?

12. A drover has 406 cows worth 30 dollars each; how much are they all worth?

13. How much will it cost to build 307 miles of railroad at 4060 dollars a mile?

14. A contractor built 604 miles of railroad at 6500 dollars a mile; how much did he get for it?

15. If it requires 720 barrels of provisions to supply an army for one day, how many barrels will be required for 365 days?

16. If one acre of land cost 9620 dollars, how much will 736 acres cost?

17. If it cost 98650 dollars to build one mile of railroad, how much will it cost to build 2809 miles?

18. There are 15 fields of corn; in each field there are 97 rows, and 256 hills in each row; how many hills are there in the 15 fields?

19. How many yards of cloth are there in 43 bales, each bale containing 72 pieces, and each piece 29 yards?

20. If a railway train goes 18 miles an hour, how far will it go in 17 days of 24 hours each?

Exercise xxiii.

Practical Problems involving the Previous Rules.

1. B bought a house for 2960 dollars, and gave for it 98 cows at 24 dollars each, and the rest in money; how much money did he pay?

2. One army contains 4575 men, and another 36 times as many, lacking 1936 men; how many men are there in the second army?

3. Mr. Peters has 2461 gallons of coal oil, Mr. Martin has 1146 gallons, and Mr. Benson has 147 times as much as both; how much has Mr. Benson?

4. A farmer sold 129 cows at 37 dollars each, and received in payment 2000 dollars; how much yet remains due?

5. B sold 76 hens at 73 cents each, 96 turkeys at 324 cents each, and received in payment 24000 cents; how much remains due?

6. A's barn cost 2485 dollars, his house cost 3 times as much, and his farm cost as much as both; what was the cost of the house? What was the cost of the farm?

7. A drover bought 36 horses at 145 dollars a head, and 96 cows at 28 dollars a head; which cost the most, and how much?

8. A's book contains 248 pages, with 2850 letters on a page, and B's contains 325 pages, with 3465 letters on a page; how many letters in A's book? How many in B's?

9. A man has 75 bags of apples, each bag containing 2 bushels; how much will be received for them, at 125 cents a bushel?

10. A farmer sold 25 firkins of butter, each firkin containing 126 pounds, and received for each pound 37 cents; how much did he receive for it all?

11. Find the product of the sum and difference of 784 and 397.

12. If 472 men cut 800 cords of wood in two days, how long would it take one man to do it?

13. A farmer sold a 129 cows at 29 dollars each, and received in payment 2300 dollars; how much yet remains due?

14. A's barn cost 175 dollars; his house cost 16 times as much, and his farm cost as much as both; what was the cost of the house? What was the cost of the farm?

15. A man bought 56 acres of land at 45 dollars an acre, and 78 acres at 62 dollars an acre, and sold the whole at 53 dollars an acre. Did he gain or lose, and how much?

16. A merchant bought 1600 barrels of flour at 7 dollars a barrel: he sold 900 barrels at 12 dollars a barrel; and the remainder at 5 dollars a barrel. Did he gain or lose, and how much?

17. If a house is worth 3250 dollars, and the farm on which it stands 3 times as much and 450 dollars more, and the stock on the farm twice as much as the house, lacking 2368 dollars; what is the value of the whole?

18. A has 4278 dollars more than B, and 1225 dollars less than C, who has 7864 dollars: and D has as much as A and B together. How much has D?

19. A man invests in trade 450 dollars at one time, at another 840 dollars, at another 1125 dollars, and at another 1640 dollars; how much must be added to these sums that the amount invested by him shall be increased three fold?

20. A man sold his house for 4500 dollars, and 250 acres of land at 75 dollars an acre; he got in payment 5000 dollars in cash, 239 cattle at 25 dollars each, and 317 sheep at 5 dollars each; how much is still due him?

21. A drover bought a herd of 33 oxen, paying as many dollars for each ox as there were oxen in the herd. He paid 500 dollars in money, and gave his note for the balance; for how much did he give his note?

Section IV.—Division.

1. Among how many boys may 20 marbles be divided, giving to each boy 5 marbles?

20 marbles.	Taking 5 marbles and giving them to the first boy, there are 15 marbles remaining,
5 "	
15 "	Taking 5 of these and giving them to the second boy, there are 10 marbles re-
5 "	maining.
10 "	Taking 5 of these and giving them to
5 "	the third boy, there are still 5 remaining.
5 "	Taking these and giving them to the fourth boy, there are none remaining.
5 "	
0 "	Hence 20 marbles may be divided among 4 boys, giving to each one 5 marbles.

If 20 marbles are divided equally among 4 boys, how many does each boy receive?

20 marbles.	Taking 4 marbles and giving each boy one, there are 16 remaining.
4 "	
16 "	Again, taking 4 more and giving each one one, there are 12 remaining, and each
4 "	boy has 2.
12 "	Again, taking 4 more and giving each
4 "	boy one, there are 8 remaining and each boy has now 3.
8 "	
4 "	Again, taking 4 more and giving each boy one, there are 4 remaining, and each
4 "	boy has 4 marbles.
4 "	Again, taking 4 more and giving each boy one there are none remaining and each boy has 5 marbles.
0	

Hence, we see that **Division is simply a short method of performing several successive subtractions of the same number.**

We might have obtained the result in a shorter way, as follows: Since 4 times 5 is 20, we see that 4 is contained in 20 five times, and 5 is contained in 20 four times.

Hence, *to find how many times one number is contained in a second, we have merely to find what number multiplied by the first will produce the second.*

2. How many times 2 horses are 6 horses?
3. How many times 3 cents are 12 cents?
4. How many times is 5 contained in 15?

Since 3 times 5 is 15, 5 is contained 3 times in 15.

5. How many times is 6 contained in 30?
6. Three dogs have 12 feet; how many feet has 1 dog?
7. How many times 4 feet are 12 feet?
8. A bush has 8 roses; how many times 2 roses has it? How many times 4 roses?
9. A house has 12 doors; how many times 3 doors has it?
10. One of the factors of 6 is 2, what is the other?
11. Divide 6 into 2 equal parts.
12. What is the half of 6?
13. How many times is 7 contained in 28?

48. When it is required to find how many times one number contains another the process is called **Division.**

49. The number to be divided is called the **Dividend.**

50. The number by which we divide is called the **Divisor.**

51. The number of times the Divisor is contained in the Dividend is called the **Quotient.**

52. When the Divisor does not go an *exact* number of times into the Dividend, the *excess* is called the **Remainder.**

53. The Remainder, being part of the Dividend, will always be of the same kind or denomination as the Dividend.

54. The **Sign of Division** is a short horizontal line, with a dot above it and another below it, thus, $\div$. It shows that the number *before* it is to be divided by the number *after* it. Thus $8 \div 2 = 4$ is read, 8 divided by 2 is equal to 4.

55. Division is frequently indicated by a line with the dividend above and the divisor below it; thus $\frac{9}{3}$ signifies that 9 is to be divided by 3.

56. Division may be divided into two cases:

1. *When the divisor does not exceed twelve.*
2. *When the divisor exceeds twelve.*

Oral Exercises.

1. If a man walks 24 miles in 6 hours, how far will he walk in 1 hour?

Distance walked in 6 hours = 24 miles.
" " 1 hour = 24 miles $\div$ 6.
= 4 miles.

2. If 1 man can do a piece of work in 32 days, how long will it take 8 men to do it?

3. If 7 yards of silk can be got for 21 dollars, how much will 1 yard cost?

4. If 27 yards of silk can be bought for 3 dollors, how many yards can be bought for 1 dollar?

5. If 3 hats cost 9 dollars, how much will 1 hat cost? How much will 7 cost? How much will 12 cost?

6. How many times 5 oranges are 50 oranges? Is the result a concrete number, or an abstract number?

7. If you can buy a lead pencil for 3 cents, how many can you buy for 24 cents?

8. How many barrels of apples at 2 dollars a barrel can be bought for 24 dollars?

9. If a man walks 3 miles an hour, how many hours will it take him to walk 18 miles?

10. A farmer divides 84 bushels of apples equally among 12 men: how many bushels does each receive?

11. 72 cents are paid for 12 eggs; how much will 1 cost at the same rate?

12. How long will it take 12 men to perform a piece of work that 1 man can do in 60 days?

13. A man planted an orchard of 120 trees, and put 10 in each row: how many rows are there in the orchard?

14. How many men at 9 dollars a month can be hired 1 month for 81 dollars?

15. If 6 barrels of flour cost 54 dollars; how much will 1 barrel cost?

Case I.

57. When the Divisor does not exceed twelve.

Ex. 1. How many times is 7 contained in 952?

Divisor.	*Dividend.*	*Quotient.*
7)	952	(136
	7	
	—	
	25	
	21	
	—	
	42	
	42	
	—	

We write the Divisor at the left, and the Quotient at the right of the Dividend and begin at the left to divide. 7 is contained in 9 hundreds 1 hundred times and a remainder. We write the 1 hundred in the Quotient, and multiply the Divisor 7 by the 1 hundred. This gives us 7 hundreds, which we write under the hundreds of the Dividend. We then subtract the 7 hundreds from the 9 hundreds and the remainder is 2 hundreds, or 20 tens. We add the 5 tens of the Dividend to these 20 tens and set down the 25 tens. 7 is contained in 25 tens 3 tens times, and a remainder We write the three tens in the Quotient, and multiply the Divisor by the 3. This gives 21 tens, which we write under the *partial* Dividend, 25 tens. We subtract, and the remainder is 4 tens or 40 units. We add the 2 units of the Dividend to these 40 units and set down the 42 units. 7 is con-

tained in 42 units 6 units times. We write the 6 units in the Quotient and multiply the Divisor by the 6. This gives us 42 units, which we subtract as before, and there is no remainder.

The working of the preceding example may be shortened as follows :—

Divisor 7) 952 Dividend.
136 Quotient.

We write the Divisor to the left of the Dividend and proceed as follows :—

7 is contained in 9, 1 time and 2 over. We place the 2 before the 5 and thus make 25. 7 is contained in 25, 3 times and 4 over. We place this 4 before 2 and thus make 42. 7 is contained in 42, 6 times.

When the *Divisor does not exceed* 12 the multiplication and subtraction are performed mentally, the quotient *only* being written down, the work being thus greatly shortened. This is called **Short Division.** The dividend may be so arranged that the units, tens, hundreds, &c., shall be divisible by the divisor ; thus :—

$$\begin{aligned} 952 &= 9 \text{ hund.} + 5 \text{ tens} + 2 \text{ units}, \\ &= 7 \text{ hund.} + 25 \text{ tens} + 2 \text{ units}, \\ &= 7 \text{ hund.} + 21 \text{ tens} + 42 \text{ units}, \\ \text{therefore, } 952 \div 7 &= 1 \text{ hund.} + 3 \text{ tens} + 6 \text{ units}, \\ &= 136. \end{aligned}$$

When all the different steps of the solution are written, the process is called **Long Division.**

Exercise xxiv.

(1)	(2)	(3)	(4)	(5)
2)36(	2)58(	2)54(	2)92(	2)96(
(6)	(7)	(8)	(9)	(10)
3)576(	3)165(	3)723(	3)873(	3)975(
(11)	(12)	(13)	(14)	(15)
4)852(	4)764(	4)932(	4)576(	4)748(

The pupil is expected to work Exercise xxiv., first by Long Division, and next by Short Division.

(16) 5)735(	(17) 5)850(	(18) 5)975(	(19) 5)745(	(20) 5)835(
(21) 6)732(	(22) 6)846(	(23) 6)924(	(24) 6)972(	(25) 6)834)
(26) 7)784(	(27) 7)798(	(28) 7)833(	(29) 7)966(	(30) 7)959(
(31) 8)896(	(32) 8)936(	(33) 8)944(	(34) 8)976(	(35) 8)992(
(36) 9)468(	(37) 9)576(	(38) 9)864(	(39) 9)738(	(40) 9)666(

Exercise XXV.

(1) 2)456	(2) 2)736	(3) 2)548	(4) 2)374	(5) 2)538
(6) 3)735	(7) 3)816	(8) 3)522	(9) 3)414	(10) 3)738
(11) 3)669	(12) 3)513	(13) 3)546	(14) 3)765	(15) 3)825
(16) 4)512	(17) 4)624	(18) 4)732	(19) 4)576	(20) 4)824
(21) 4)736	(22) 4)816	(23) 4)972	(24) 4)608	(25) 4)436
(26) 5)615	(27) 5)735	(28) 5)645	(29) 5)785	(30) 5)840
(31) 5)815	(32) 5)935	(33) 5)780	(34) 5)765	(35) 5)880
(36) 6)834	(37) 6)648	(38) 6)654	(39) 6)774	(40) 6)864

(41)	(42)	(43)	(44)	(45)
6)1476	6)3336	6)2514	6)3654	6)7338
(46)	(47)	(48)	(49)	(50)
7)2569	7)4732	7)8456	7)9359	7)9870
(51)	(52)	(53)	(54)	(55)
8)7256	8)3656	8)7576	8)29352	8)111032
(56)	(57)	(58)	(59)	(60)
9)8892	9)3978	9)2565	9)63288	9)67356

Exercise xxvi.

Practical Problems.

1. At 6 cents each, how many oranges can be bought for 354 cents?

2. At 2 dollars a day, how many days' work can I hire for 346 dollars?

3. How many pounds of rice at 4 cents a pound can I buy for 3672 cents?

4. In three feet there is 1 yard; how many yards are there in 693 feet?

5. If 8 men can dig 768 rods of ditch in 3 weeks, how many rods can 1 man dig in the same time?

6. If 7 yards of cloth cost 637 cents, what will 1 yard cost?

7. If 9 men dig 135 bushels of potatoes in 1 day, how many bushels can 1 man dig in 1 day?

8. When 7 is multiplied by a certain number the produce is 861, what is the number?

9. If 6 bins of equal size are exactly filled by 36312 bushels of grain, how much does each bin hold?

10. If 7 men can cut 56 cords of wood in 4 days, how much can 1 man cut in the same time?

Ex. 2. Divide 70268 by 7.

Divisor 7) 70268 Dividend.

10038 Quotient. 2 Remainder.

In this example we say 7 is contained in 7 ten thousands, 1 ten thousand times and no remainder. We put down this 1 in the ten thousands' places. 7 is not contained in 0 thousands. We put a 0 in the thousands' place. 7 is not contained in 2 hundreds. We again write a 0 in the hundreds' place. 7 is contained in 26 tens, 3 tens times and 5 over. We write the 3 in the tens' place, 7 is contained 8 times in 58 units and 2 over. We write 8 in the units' place and indicate the division of the 2, thus, $\frac{2}{7}$; this is annexed to 10038, thus, $10038\frac{2}{7}$.

58. *PROOF.*—Multiply the Quotient by the Divisor, and to the Product add the Remainder, if any, and if the result is the same as the Dividend the work is likely to be correct.

Exercise xxvii.

Divide,

1. 6532 by 3.
2. 11236 by 9.
3. 57636 by 6.
4. 11485 by 7.
5. 98537 by 8.
6. 345246 by 5.
7. 1680245 by 4.
8. 3432026 by 6.
9. 6216563 by 8.
10. 7295849 by 10.
11. 16779120 by 12.
12. 37000305 by 5.
13. 5767692 by 7.
14. 56464237 by 9.
15. 46626289 by 11.
16. 3523360 by 6.
17. 160590736 by 8.
18. 370370480 by 10.
19. 101650247 by 12.
20. 51088982 by 7.
21. 67320837 by 9.
22. 30040526 by 11.
23. 106131923 by 12.
24. 740048200 by 8.
25. 45603875 by 10.
26. 336384072 by 9.

Exercise xxviii.

Practical Problems.

1. When flour is worth 8 dollars a barrel, how many barrels could be brought for 3456 dollars?

2. If 7 casks of sugar weigh 8792 pounds, what is the average weight of each cask?

3. A father dying left an estate of 37356 dollars to be divided equally among his wife, his two sons and his three daughters; what was the share of each?

4. Five men bought a horse for 160 dollars; they hire him out at 4 dollars a day for 21 days, and sell him for 120 dollars; how much will each one gain?

5. A grocer bought 15 barrels of flour for 100 dollars; he sold it so as to gain 20 dollars; how much did he receive per barrel?

6. How long will it take two boys, starting at the same place, and travelling in opposite directions, to be 29076 rods apart, if one goes 5 and the other 7 rods in a minute?

7. If 66 apples are divided equally among 5 boys, how many does each boy receive?

8. If 4 sacks of coffee weigh 523 pounds, what is the weight of each?

9. If 626 dollars are divided equally among 5 men what will be the share of each?

10. In one week there are 7 days; how many weeks are there in 365 days?

11. John, James and William have altogether 756 marbles, which they wish to divide equally; what will be the share of each?

12. A man has 4 equal lots of land, containing in all 2759 acres; how many acres are there in each lot?

13. If 9 car-loads of freight weigh 144712 pounds, what is the weight of each car-load?

14. If 8 waggons carry 1381 bricks, how many bricks can be carried in one wagon?

Case II.

59. When the Divisor is greater than Twelve.

Ex. 3. Divide 4839 by 17.

Divisor.	*Dividend.*	*Quotient.*	
17)	4839	(284	* 1— 17
	34		2— 34
	——		3— 51
	143		4— 68
	136		5— 85
	——		6—102
	79		7—119
	68		8—136
	—		9—153
	11 Remainder.		

Since 17 is not contained in 4 thousands any thousand times, we unite the 4 thousands to the 8 hundreds, making 48 hundreds; 17 is contained in 48 hundreds 2 hundred times. We set down 2 as the first figure in the quotient, then multiply 17 by 2, and subtract the product 34 from 48. The remainder is 14. To this remainder we annex the 3 tens of the dividend, making 143 tens; 17 is contained in 143 tens 8 tens times. We set down 8 as the next figure of the quotient, then multiply 17 by 8, and subtract the product 136 from 143. The remainder is 7; to this remainder annex the next figure of the dividend, and continue as before.

Ex. 4. Divide 74198 by 37.

37)	74198	(2005
	74	
	———	
	198	
	185	
	———	
	13	

*Let the pupil, before commencing the operation of dividing, construct a table by multiplying the divisor by each number successively up to 9 in the manner indicated in the example. The proper quotient will then be apparent on inspection. The different products may be found more easily by addition. Thus, the *third* product is found by adding the first and second; the fourth product, by adding the first and third, and so on.

In this example we find there is no remainder on subtracting 74 from 74, and on bringing down 1, the third figure of the dividend, 37 is not contained in it; we therefore write 0 as the second figure of the quotient. When we bring down 9, the next figure of the dividend, 37 is not contained in 19; we therefore write another 0 as the third figure of the quotient. When we bring down 8, the last figure of the dividend, 37 is contained in 198, 5 times, and we go on as before.

Note.—*For every figure of the dividend brought down* **one figure** *must be written in the quotient.*

The proof is the same as in *Short Division.*

284
17

1988
284

4828 Product
11 Remainder

4839 Dividend

or thus:—Add the remainder and the several partial products; thus:—

3400 1st Partial Product.
1360 2nd " "
68 3rd " "
11 Remainder.

4839 Dividend.

Exercise xxix.

Divide

1. 764 by 31.
2. 367 by 41.
3. 987 by 53.
4. 4567 by 61.
5. 2936 by 74.
6. 38271 by 65.
7. 29781 by 56.
8. 71847 by 76.
9. 67654 by 122.
10. 39298 by 801.
11. 80157 by 346.
12. 466281 by 936.
13. 159750 by 425.
14. 539902 by 239.
15. 999999 by 198.
16. 2802690 by 990.

17. 8991207 by 1449.
18. 9672160 by 1560.
19. 6192138 by 1653.
20. 3515772 by 1736.
21. 9876480 by 1976.
22. 24197460 by 2492.
23. 8231505 by 1905.
24. 13896225 by 2975.
25. 16084440 by 5058.
26. 23103465 by 6391.
27. 18356508 by 16074.
28. 572105376 by 78617.
29. 344943192 by 134376.
30. 1806147420 by 35805.

Exercise xxx.

Practical Problems.

1. There are 24 hours in a day; how many days are there in 1032 hours?

2. If a man walks 25 miles in a day, how long will it take him to walk 950 miles?

3. Sound moves 37060 feet in 34 seconds; how far will it move in 1 second?

4. A drover bought 23 head of cattle for 736 dollars; what was the price per head?

5. In 1 year there are 52 weeks; how many years are there in 6708 weeks?

6. If 75 shares of bank stock sell for 9225 dollars, what is the price per share?

7. A man bought a farm of 524 acres for 24104 dollars; what was the average price per acre?

8. How many bales could be made out of 281765 pounds of cotton, allowing 517 pounds to the bale?

9. If a steamship sails 5836 miles in 17 days, what would be the average daily distance?

10. A flour barrel holds 196 pounds of flour; how many barrels would it take to hold 406700 pounds?

ABBREVIATED PROCESS IN LONG DIVISION.

Case I.

60. To divide by a composite number.

61. A Composite Number is one which may be produced by multiplying together two or more numbers, neither of which is 1. Since $16 = 8 \times 2$, 16 is a *composite* **number.**

Ex. 5. Divide 8769 by 42.

7) 8769

6) 1252 and 5 units over = 5

208 " 4 sevens over = 28

Remainder = 33

Since the factors of 42 are 7 and 6, we divide by these factors in succession. First, dividing by seven we obtain 1252 for quotient and 5 for remainder. This is 5 units. We then divide the quotient by six and obtain 208 for quotient and 4 for remainder. This is 4 groups of 7 units each or 28 units. The remainder is, therefore, 28 units + 5 units = 33 units.

Hence, The true remainder is found by multiplying the last remainder by the first divisor and adding to the product the first remainder.

Exercise xxxi.

Divide

1. 46827 by 27,
2. 87468 by 64.
3. 97648 by 63.
4. 13853 by 45.
5. 8642396 by 35.
6. 7308216 by 49.
7. 8742 by 25.
8. 66842 by 96.
9. 87648 by 81.
10. 419421 by 99.
11. 339240 by 132.
12. 806345 by 144.

Case II.

62. To Divide when there are ciphers at the right of the Divisor.

Ex. 6. Divide 8593 by 700.

The divisor, 700, may be resolved into the factors 7 and 100. We first divide by the factor 100 by cutting off two figures at the right, and get 85 for the quotient and 93 for the remainder. We then divide the quotient, 85, by the other factor, 7, and obtain 12 for the quotient and 1 for a remainder. The last

7,00) 85,93

12 and 193 rem.

remainder, 1, being multiplied by the divisor, 100, and 93, the first remainder, added, we obtain 193 for the true remainder.

Hence, *To divide when there are ciphers at the right of the divisor, we cut off the ciphers from the divisor and the same number of figures from the right of the dividend; we then divide the remaining figures of the dividend by the remaining figures of the divisor and prefix the remainder to the figures cut off, and the result will be the true remainder.*

Exercise xxxii.

Divide

1. 725 by 30.
2. 7642 by 60.
3. 8642 by 700.
4. 97861 by 300.
5. 72369 by 90.
6. 94678 by 80.
7. 3786 by 1700.
8. 21500 by 3600.
9. 378751 by 12300.
10. 984721 by 6400.
11. 1684273 by 2500.
12. 9486279 by 15000.

Exercise xxxiii.

Practical Problems.

1. In a yard there are 36 inches; how many yards are there in 3888 inches?

2. There are 60 minutes in an hour; how many hours are there in 3900 minutes?

3. There are 16 ounces in a pound; how many pounds are there in 1968 ounces?

4. How many pounds of beef at 18 cents a pound can be bought for 540 cents?

5. There are 64 pints in a bushel; how many bushels are there in 2688 pints?

6. A farmer sold 24 horses for 5640 dollars: how much did he receive apiece for them?

7. There are 25 pounds in a quarter; how many quarters are there in 34450 pounds?

8. How many bushels of oats, at 56 cents a bushel, can be bought for 13272 cents?

9. If 48 acres of land produce 2064 bushels of corn, how much will be produced from one acre?

10. If a man travels 2052 miles in 54 days, what is the average rate of travel per day?

63. If any *three* of the four numbers, that form the Divisor, Dividend, Quotient and Remainder be given, we can find the *fourth*.

1. Let Divisor, Dividend, and Quotient be given. Multiply the Divisor by the Quotient, subtract the result from the Dividend, and we have the Remainder.

2. Let Divisor, Quotient, and Remainder be given. Multiply the Divisor by the Quotient, add the Remainder to the result, and we have the Dividend.

3. Let Divisor, Dividend, and Remainder be given. Subtract the Remainder from the Dividend, divide the result by the Divisor, and we have the Quotient.

4. Let Quotient, Dividend, and Remainder be given. Subtract the Remainder from the Dividend, divide the result by the Quotient, and we have the Divisor.

Exercise xxxiv.

1. What number divided by 75 will give a quotient of 117 and remainder of 39?

2. What number must be taken from 9765 so that it may be exactly divisible by 132?

3. Of what number is 183 both divisor and quotient?

4. What number larger than 216 will divide 75168 without a remainder?

5. What number must be added to 38172 so that it may be exactly divisible by 379?

6. The answer to a question in Multiplication is 1401336 the multiplicand is 5163; what is the multiplier?

7. If the quotient is 5000 when the divisor is 2001 and the remainder 100, what is the dividend?

8. What number divided by 528 will give 36 for quotient and leave 44 as a remainder?

9. If the dividend be 784622 and the quotient be 4044, what is the divisor and the remainder?

10. If the quotient be 194, the divisor 4044, and the remainder 87, what is the dividend?

Exercise xxxv.

Practical Problems involving the Previous Rules.

Ex. 1. A carpenter can earn 45 dollars a month; his expenses are at the rate of 24 dollars a month. He wishes to purchase a lot of ground which contains 19 acres, and is held at 42 dollars per acre; in what time may he save enough to make the purchase?

He saves 45 dollars − 24 dollars = 21 dollars a month.

The lot will cost 19×42 dollars = 798 dollars;

then the number of months in which he can save enough to purchase the lot is $798 \div 21 = 38$.

1. A farmer bought land from A at 60 dollars an acre, and the same quantity from B at 85 dollars an acre. The whole amounted to 53215 dollars; how many acres did he buy from each?

2. A merchant sold a piece of cloth containing 45 yards, another piece containing 57 yards, and another containing 63 yards, at 14 dollars a yard; what did the whole amount to?

3. A man left 2535 dollars each to his four children, but one of them dying the three remaining children divided the money equally among them; how much did each receive?

4. A man earns 25 dollars a week, and spends 12 dollars a week: he saves 195 dollars; how many weeks does he work?

5. A farmer has 24 cows and 93 sheep, worth 1521 dollars; if the sheep are worth 5 dollars each, how much is each cow worth?

6. If 28 men earn 7946 cents in a day, and 25 boys earn 5450 cents in a day, how much more does one man earn in a day than one boy?

7. How many barrels of flour at 6 dollars a barrel are equal in value to 1100 tons of coal at 9 dollars a ton?

8. If a mechanic earns 52 dollars a month, and his expenses are 34 dollars a month, how long will it take him to pay for a farm of 36 acres, worth 12 dollars an acre?

9. A clerk's salary is 1200 dollars a year; he pays 5 dollars a week for board, 2 dollars a month for car fare, and his other expenses amount to 1 dollar a day; how much can he save in a year?

10. Mr. Jones bought a farm of 100 acres at 75 dollars an acre, 2200 dollars to be paid down, and the remainder in five equal yearly instalments; what must he pay each year?

11. A man has 13 piles of wood, each containing 25 cords, and each cord 128 cubic feet; how many cubic feet of wood has he?

12. A man exchanged 159 cords of wood at 5 dollars a cord, for a horse valued at 144 dollars, and the balance in sheep at 3 dollars each; how many sheep did he receive?

13. A merchant balancing his accounts found that he had on hand merchandise worth 175 dollars, and cash amounting to 2570 dollars; he had lost by bad debts 250 dollars, and owed 525 dollars; if his original capital was 2000 dollars, what had he gained?

14. A cistern containing 13500 gallons is filled by two pipes, one discharging 250 gallons an hour, and the other 300 gallons; but, by a leak in one of the pipes, 100 gallons are lost in an hour; if the cistern is empty, how long will it take to fill it?

Ex. 2. If 3 pounds of coffee cost 30 cents, what will 8 pounds cost?

The cost of 3 pounds of coffee = 30 cents;

" 1 pound " $= \frac{30}{3} = 10$ cents;

" 8 pounds " $= 8 \times 10$ cents $= 80$ cents.

15. What will 15 slates cost, if 5 slates cost 80 cents?

16. If 4 trees cost 72 cents, what will 3 trees cost?

17. If 6 barrels of flour cost 48 dollars, what will 7 barrels cost?

18. What will be the cost of 16 cords of wood if 4 cords cost 24 dollars?

19. If 15 yards of cloth cost 75 dollars, what will 20 yards cost?

20. If 7 pounds of beef cost 56 cents, what will 5 pounds cost?

21. If 12 men can earn 36 dollars in a day, how much can 4 men earn in the same time?

22. If 28 acres of land cost 4480 dollars, how much will 43 acres cost at the same rate?

23. In 52 years there are 18980 days; how many days are there in 65 years?

24. Twenty-five barrels of flour weigh 4900 pounds; what is the weight of 36 barrels?

25. If you can buy 765 yards of cloth for 51 dollars, how many can you get for 376 dollars?

26. If 38 acres of land cost 11172 dollars, how many acres can be bought for 107310 dollars?

27. If 13 houses cost 16250 dollars, what will 25 houses cost?

28. If 17 horses cost 1802 dollars, how much will 9 horses cost?

29. Fifteen men can husk 1095 bushels of corn in a day; how many bushels can 27 men husk?

Ex. 3. If 7 men do a piece of work in 36 days, in how many days can 28 men do it?

Time for 7 men to do the work = 36 days;

" 1 man " " $= 7 \times 36$ days;

" 28 men " " $= \frac{7 \times 36}{28} = 9$ days.

30. If 15 workmen can do a piece of work in 25 days, in what time can 25 men do the same?

31. A field can be mowed by 40 men in 9 days; in how many days would it be finished by 30 men?

32. If 16 men can build a house in 20 days, how long would it take 10 men to build it?

33. If 19 men can finish a work in 437 days, how long will it take 23 men to do the same work?

34. If 18 horses can cart away the earth from a cellar in 75 days, in how many days would 27 horses do this work?

35. Ten men engage to build a house in 63 days, but 3 of them being taken sick, how long will it take the rest to build the house?

36. If 6 carpenters can build a house in 72 days, how long would it take 9 carpenters to build the same?

37. How long will it take 40 men to build a wall, if 12 men can do it in 20 days?

38. How long will it take 9 men to do the same amount of work that 6 men can do in 15 days?

39. How long will 19 men take to do a piece of work which requires 17 men 133 days to do?

Ex. 4. If 30 men build a wall in 18 days, how many men will be required to do it in 12 days?

Men required to build the wall in 18 days = 30 men;

" " " " 1 day = 18×30 men;

" " " " 12 days = $\frac{18 \times 30}{12} = 45$ men.

40. If 4 men can dig a garden in 7 days how many men would be required to dig it in 1 day?

41. If 28 men can mow a field of grass in 15 days, how many men will be required to mow it in 4 days?

42. If 7 men can reap a field of wheat in 18 days, how many men would be required to do the same work in 6 days?

43. A piece of work was to have been performed by 144 men in 36 days, but a number of them having been discharged, the work was performed in 48 days; how many men worked?

44. If 20 men can perform a piece of work in 15 days, how many men will it take to do it 12 days?

45. How many men in 26 days can perform the same amount of work that 39 men can do in 76 days?

46. A drain is dug by 49 men in 96 days; now many men would have been required to dig it in 84 days?

47. If 8 workmen can build a wall in 27 days, how many workmen would be required to build it in 3 days?

48. If 100 workmen can perform a piece of work in 12 days, how many men are sufficient to perform the work in 8 days?

49. A gentleman met a number of beggars, and relieved 9 of them by giving 25 cents to each one; how many would he have relieved for the same sum had he given them only 15 cents apiece?

EXAMINATION PAPERS.

I.

1. Define the following terms: Unit, Number, Notation, and Numeration.

2. Add together four millions twenty thousand and seventy-nine, twelve millions two thousand and seven, and one million and five thousand, and subtract 16538107 from the sum.

3. Find the remainder after subtracting the numbers 44444, 9999, 666, 77, 1, in succession from 1000000.

4. Add together the sum, difference, product, and quotient of the two numbers 825 and 9318375.

5. I bought a farm of 136 acres for 8568 dollars, and sold 93 acres of it at 75 dollars an acre, and the remainder for what it cost; how much did I gain by the bargain?

II.

1. Explain the meaning of the following terms, and give an example of each: Subtrahend, Multiplicand, Product, Divisor, Quotient, Remainder.

2. Find the sum of the following numbers, and express the result in words: 1234567, 8765433, 6894703, 3105297, 5712843, and 4187157.

3. What is the difference between the aggregate of 1050, 325, 1769, 150801, and a million? Show that the same difference is obtained by taking one of the numbers from a million another from the remainder, and so on for the rest of the numbers.

4. Express MMDCXCIX and CCCXXIX in the ordinary numerical characters: find their product, and express the result in Roman characters.

5. How many bushels of wheat, at 125 cents per bushel, should be exchanged for 250 pounds of sugar, at 8 cents per pound?

III.

1. From 7503 take 871, and explain the process of "borrowing and carrying" in the common rule of subtraction.

2. How may the process of subtraction be verified? Give an example.

3. By how much does the sum of the numbers 2718281828 and 3141592653 exceed their difference?

4. What arithmetical operation bears the same relation to subtraction that multiplication bears to addition?

5. Bought a farm for 35380 dollars, and having made improvements valued at 3420 dollars, I sold one-half of it for 21750 dollars, at 75 dollars an acre; how many acres did I purchase, and at what price per acre?

IV.

1. What is the object of division? Show that it may be considered a shortened subtraction.

2. What are the *factors* of a number?

3. If division by a composite number be performed by successively dividing by its factors, show how the complete remainder may be found. Ex. 1437281 divided by 105.

4. How much can a man earn in 114 days, if he can earn 48 dollars in 24 days?

5. A man bought a number of sheep at the rate of 3 for 18 dollars; how many did he buy for 3648 dollars?

V.

1. What is multiplication? Show by an example that it is a short method of performing addition.

2. Show by an example that two or more factors will give the same product in whatever order they are multiplied.

3. How many times must 1874 be added to itself to make a total of 163038?

4. The product of 75 by 43 is 3225; how much must be added to it to obtain the product of 77 by 43?

5. A drover bought 79 oxen at 42 dollars each; he sold 25 at 40 dollars each; for how much per head must he sell the rest so as to gain 544 dollars on the whole transaction?

VI.

1. Given the divisor, quotient, and remainder, how is the dividend found?

2. I bought a farm of 150 acres for 12000 dollars; I sold 29 acres at 95 dollars an acre, 75 at 112 dollars an acre, and the rest at 96 dollars an acre; what did I gain by this transaction?

3. What number is that, which being multiplied by 15, the product divided by 16, the quotient multiplied by 7, 35 subtracted from the product, the remainder divided by 10, and 52 subtracted from the quotient, the remainder is 18?

4. I bought a farm for 6480 dollars, and after spending 890 dollars on improvements on it, I sold one half of it for 4050 dollars at 45 dollars an acre; how many acres did I buy, and at what price per acre?

5. If 16 men can perform a piece of work in 36 days, in how many days can they do it with the aid of 8 more men?

VII.

1. Explain why in addition of numbers the operation is begun at the units' place. Is this necessary? Illustrate by an example.

2. A person willed his property to his three children. To the youngest he gave 2149 dollars, to the second 3 times as much, and to the eldest 5 times as much as to the second; find the value of the property.

3. Two ships 3120 miles apart approach each other, the one sailing at the rate of 146 miles a day, and the other at 127 miles; how far will they be apart at the end of 9 days?

4. John found a bagful of coins. On counting them he found there were 5 cent pieces, 10 cent pieces, and 20 cent pieces in it, and the same number of each; how many of each were there, if the whole amounted to 8645 cents?

5. A gentleman dying disposed of his property, worth 53175 dollars, as follows: he left 1500 dollars to a church; 4 times this sum to a college; and he divided the remainder equally among his 5 sons and 2 daughters; what was the share of each child?

VIII.

1. If a man has a salary of 2100 dollars a year, and spends 4 dollars a day, how much will he save in 5 years, allowing 365 days in a year?

2. What number must be taken 708 times from 688953 so as to leave 69 for remainder?

3. A drover bought 12 head of cattle at 22 dollars each; 9 head at 25 dollars each; and 4 head at 32 dollars each; at how much per head must he sell them so as to gain 158 dollars?

4. Three boys go picking berries and agree to divide the proceeds equally: the first picks 15 quarts and sells them at 13 cents a quart; the second picks 16 quarts and sells them at 12 cents a quart; and the third picks 12 quarts and sells them at 18 cents a quart; find what each one gets.

5. Two travellers, *A* and *B*, meeting on a journey, found that the whole distance both had travelled was 2145 miles, and that *B* had gone 217 miles further than *A*; how far had each travelled?

CHAPTER II.

Canadian Money.

64. Canadian Money is the legal currency of the Dominion of Canada. It is composed of *dollars, cents*, and *mills*. The **dollar** is the unit, and is denoted by the symbol $.

65. 10 mills = 1 cent.
100 cents = $1.

66. Dollars are separated from cents, in writing, by a *point*. Thus $6.75 is read six dollars and seventy-five cents. Any number of cents less than ten, when written with dollars, occupies the second place to the right of the *point*, and the first place is occupied by a cipher; thus, $4.05 is read four dollars and five cents. The *mill* is one-tenth of a cent, and is written one place to the right of the cents; thus, $3.755 is read 3 dollars, 75 cents, and 5 mills.

67. The present silver coins of the Dominion are the fifty-cent piece, the twenty-five cent piece, the ten-cent piece, and the five-cent piece. The only copper coin is the one-cent piece.

NOTE.—The *mill* is not coined; it is used only in computation. When the final result of a business computation contains mills, if 5, or more, they are reckoned 1 cent, and if less than 5 they are rejected.

68. Since numbers expressing mills, cents, and dollars increase from right to left in the same manner as the numbers with which we have been dealing, they may be added, subtracted, multiplied, and divided in the same manner.

Exercise XXXVI.

Read the following:

$1.15.	$14.25.	$21.50.	$.243.
$3.24.	$26.00.	$107.16.	$.803.
$11.17.	$18.05.	$107.60.	$8.03.
$19.30.	$25.07.	$100.70.	$6.003.

Write in figures:

1. Five dollars and twenty-five cents; eighty-seven dollars and forty cents.

2. Seventy dollars and sixty-seven cents; two dollars and four cents.

3. Ninety dollars and nine cents; one hundred and one dollars and ten cents.

4. One hundred and twenty-nine dollars and one cent; nine hundred dollars and ninety cents.

5. One thousand dollars; one thousand and seven dollars and three cents.

6. Five thousand three hundred dollars and forty-three cents.

7. Twenty-three thousand and five dollars; forty thousand dollars, forty cents, and five mills.

8. Five thousand dollars and five cents; five hundred thousand and nine dollars and thirty-seven cents.

9. Four hundred and eighty thousand dollars; five hundred thousand five hundred dollars, fifty cents and seven mills.

10. One million dollars; one million, one thousand and one dollars, one cent and one mill.

Reduction.

Oral Exercise.

1. How many cents are there in $3?
2. How many cents are there in $2?
3. How many cents are there in $3.16?

4. How many cents are equal to a five-dollar bill?

5. How many cents are equal to a dollar bill and 25 cents?

6. How many cents are there in a half dollar and a quarter dollar?

7. How many cents are there in one dollar and a half?

8. How many ten-cent pieces are there in $4?

9. How many cents are equal to 2 five-dollar bills?

10. How many five-cent pieces are there in $2?

69. Reduction is the process of changing the denomination or name of a number without changing its value.

Ex. 1. How many cents are there in $3.29?

Since $1 = 100 cents;

$3 = 3 × 100 cents or 300 cents;

300 cents and 29 cents make 329 cents;

therefore, $3.29 cents = 329 cents.

Hence, *In reducing a number of dollars and cents to cents, we simply remove the point.*

Ex. 2. How many dollars are there in 6904 cents?

Since 100 cents = $1;

1 cent = $\$\frac{1}{100}$;

6904 cents = $\$\frac{6904}{100}$ = $69.04.

For when 6904 cents are divided by 100 the quotient is 69, and the remainder 4 cents.

Hence, *In reducing cents to dollars the point must be placed two places from the right.*

Exercise xxxvii.

Reduce to cents

1. $5;	$7.36;	$17.04.
2. $29.18;	$141.36;	$200.09.
3. $361.07;	$500.75;	$1000.10.
4. $1875.63;	$3647.29;	$76841.06.
5. $20063.07;	$141368.79;	$10010010.01.

Reduce to dollars and cents

6.	368 cents;	700 cents;	1236 cents.
7.	3641 cents;	7008 cents;	910988 cents.
8.	54168 cents;	500709 cents;	684007 cents.
9.	300041 cents;	280014 cents;	340001 cents.
10.	2900009 cents;	7010013 cents;	10000091 cents.

Addition.

Oral Exercises.

1. A book cost \$1.25, and a slate 50 cents; how much did they both cost?

2. A pound of tea cost \$1, a pound of coffee 25 cents, and a ham \$1.75; what was the total cost?

3. If I pay \$1.20 for a turkey, \$1.15 for a goose, and 60 cents for butter; how much did I pay for all?

4. Bought a pig for \$6, a bag of flour for \$4, and a cord of wood for \$7.50; how much did I pay for all?

5. Paid 90 cents for paper, 10 cents for pens, and \$1.25 for a book; how much did I pay for all?

6. A book cost 90 cents, a pen holder 10 cents, and a slate 35 cents; how much do they all cost?

Ex. 3. Add together \$7.37, \$29.78, \$0.29, \$187.04, and \$500.

\$ 7.37	As we must add things of the same kind, we write dollars under dollars and cents under cents, letting the points range in a straight line. Then regarding the dollars and cents as so many cents, we add as in simple numbers and place the point in the sum two places from the right to reduce the cents to dollars.
29.78	
0.29	
187.04	
500.00	
\$724.48	

Exercise xxxviii.

(1)	(2)	(3)	(4)
\$71.36	\$184.36	\$1843.21	\$105.20
169.08	769.28	978.89	110.00
208.72	41.07	36.07	409.05
714.39	869 36	362.48	1000.65

5. A farmer receives $15.37 for a cow, $75 for a horse, $3.13 for some potatoes, and $5.55 for some poultry; how much does he receive in all?

6. Sold some velvet for $3.33, broadcloth for $18.75, silk for $12.50, muslin for $5.40, carpeting for $30.05, a shawl for $12.25; what is the amount of the bill?

7. If a house costs $3487.45; repairs, $53.37; painting, $119.23; furniture, $1563.39; moving, $9; what was the whole cost?

8. A lady gives 25 cents for needles, $17.50 for a dress, $2.63 for trimmings, $1.50 for a cap, and 12 cents for thread; how much does she lay out?

Subtraction.

Oral Exercises.

1. John bought a book for $1.50 and sold it for $1.75; how much did he gain?

2. A merchant bought goods for $4.75, and sold them for $6; how much did he gain?

3. John had $10; he paid $2.50 for books, and $1.50 for a satchel; how much money has he left?

4. Mary had $1.25; she paid 75 cents for some ribbons, and 25 cents for car tickets; how much has she left?

5. Bought some rice for 60 cents, some sugar for 45 cents, and some tea for $1; how much change should I get from a five-dollar bill?

6. Bought a horse for $120, a saddle for $15, and sold both for $150; what was my gain?

7. I bought a pound of rice for 8 cents, crackers for 15 cents, raisins for 18 cents, candy for 10 cents; how much change should I get back if I gave the clerk $1.00?

Ex. 4. John owes $137.35 and pays $29.17; how much does he still owe?

$$\begin{array}{r} \$137.35 \\ 29.17 \\ \hline \$108.18 \end{array}$$

Writing dollars under dollars and cents under cents, we regard the dollars and cents as so many cents, and subtract as in simple numbers. We then place the point two places from the right of the remainder to reduce the cents to dollars.

Exercise xxxix.

(1)	(2)	(3)	(4)
$104.36	$76.14	$200.00	$782.36
9.78	17.39	156.81	189.75

5. A man has $10000; he buys a house worth $4829.36; how much money has he remaining?

6. John has $17.21, James has $41; how much has James more than John?

7. My salary is $1000 a year; I pay for rent $150, for groceries $325.40, for butter $60.30, for dry goods $127.63, and for other expenses $75.60; how much do I save?

8. A man worth $10000 gave away $956.38, and lost $1127.82; what was he then worth?

9. If a lady gives 12 cents for ink, 63 cents for pens, $13.30 for books, and $1.87 for paper, how much change must she get from a twenty-dollar bill?

10. Bought $75 worth of hay, and $25.25 worth of corn; paid $49.88; how much is still due?

11. I paid $1637.25 for a farm, $3675.25 for building a house, and $2896.87 for building a barn; I sold my property for $13000; how much did I gain?

12. I paid $246.75 for a horse, $325.45 for a mule, $42.95 for an ox, $37.50 for a cow; I sold them all for $603.50; what was the loss?

Multiplication.

Oral Exercises.

1. What will 10 pounds of fish cost at 12 cents a pound?

2. What will three pair of boots cost at $5.25 a pair?

3. If I earn $10.50 in 1 week, how much can I earn in 2 weeks?

4. Bought 2 hats at $1.25 each, and 3 collars at 25 cents each; how much did I pay for them?

5. Thomas earns 75 cents a day; his expenses are 52 cents a day; how much does he save in seven days?

6. A man bought 4 bushels of wheat at $1.12, and sold the flour for $5; how much did he gain?

7. Bought 5 barrels of flour at $8.50 a barrel, and 6 bushels of wheat at $1.25 a bushel; what was the cost of both?

8. What is the cost of 2 pair of chickens at 75 cents a pair, and 5 pair of ducks at 60 cents a pair?

9. Bought 5 pounds of coffee at 35 cents a pound, and 12 pounds of ham at 22 cents a pound; how much change did I get from a five-dollar bill?

Ex. 5. Multiply $78.39 by 8.

$78.39
8
$627.12

We regard the dollars and cents as so many cents and multiply as in Simple Multiplication, and then we place the point two places from the right of the product, to reduce the cents to dollars.

Exercise xl.

	(1)	(2)	(3)	(4)
Multiply	$78.37	$247.16	$48.75	$781.36
By	5	6	19	125

5. A farmer sold 175 acres of land at $37.50 an acre, how much did he get for the land?

6. A miller sold 525 barrels of flour at $6.71 a barrel; how much did he receive for all of it?

7. What will 42 calves cost at $3.75 each.

8. At 37 cents each, what will 75 geese cost?

9. What will 890 cords of wood cost at $3.78 a cord?

10. What will be the cost of 14 yards of black silk at $1.20 a yard?

11. If a boy's wages are $4.75 a week, how much will he earn in a year, or 52 weeks?

12. If a clerk earns $8 a week, and spends $4.75 a week, how much will he lay by in a year?

13. What will it cost six persons to board for a year at the rate of $5.75 each for a week?

14. What is the value of 17 chests of tea, each weighing 59 pounds, at $0.72 a pound?

15. A merchant sold 15 barrels of pork, each weighing 200 pounds at 12 cents a pound; what did he receive?

16. A lady goes to market with 10 dollars; she buys 6 dozen eggs at 27 cents, 7 pounds of meat at 16 cents, and 3 bushels of potatoes at $1.25; how much money has she remaining?

17. A drover bought 95 cows at $37.25 each, and sold them at $40 each; how much did he make?

18. Mr. Good bought 15 hogsheads of molasses, containing 63 gallons each, at 65 cents a gallon, and sold it at $1.10 a gallon; what was his gain?

Division.

Oral Exercises.

1. If 7 hens cost $3.57, what will one cost?

2. At 5 cents each, how many oranges can I buy for $1?

3. I paid $18.24 for 6 weeks' board; how much did I pay a week?

4. At 6 cents each, how many lemons can I buy for $3.72?

5. If 4 hats cost $5, what will 7 such hats cost?

6. A yard of calico is worth 12 cents; if I buy 15 yards and give a two-dollar bill in payment, how many oranges at 5 cents each can I buy with the change?

7. If a barrel of flour costs $6.25, how many barrels can be bought for $50?

8. At the rate of 15 cents a dozen, how many dozen buttons can be bought for $3?

9. If I buy 17 pounds of sugar at 10 cents a pound, how many oranges at 5 cents each can I get for the change due me from a five-dollar bill?

10. A yard of calico is worth 9 cents; how many yards can I get for 10 dozen of eggs, worth 18 cents a dozen?

11. If I trade 6 pounds of butter at 20 cents a pound, and 10 pounds of lard at 12 cents a pound, for sugar at 12 cents a pound, how many pounds of sugar do I get?

Ex. 6. Divide \$639.75 by 5.

5) \$639.75
\$127.95

We regard the dollars and cents as so many cents and divide as in Simple Division. Then we place the point in the quotient to separate the dollars from the cents.

Ex. 7. When potatoes are worth \$1.25 a bag, how many bagfuls can be bought for \$46.25?

125) 4625 (37
375
875
875

We are required to find how often \$1.25 is contained in \$46.25. We regard \$1.25 as 125 cents and \$46.25 as 4625 cents, and then we divide in the usual way.

Exercise xli.

(1)	(2)	(3)	(4)
6) \$76.32	7) \$149.59	8) \$145.36	9) \$237.06

5. If a person spends \$410.28 in a year how much is that in a week, allowing 52 weeks to a year?

6. Divide \$2117.71 equally among 35 families, and find the share of each.

7. A man pays for some land \$400 cash and \$192.80 in produce. If there were 57 acres, how much does the land cost him per acre?

8. How many sheep can be bought for \$302.95 at \$4.15 each?

9. If 93 oranges cost $5.58, what will 37 cost?

10. I bought a house at $3453, and paid for it in instalments of $575.50 each; how many payments did I have to make?

11. William earned $3.25 a day and paid 75 cents for board; in how many days would he save $912.50?

12. A merchant received $853.25 for a case of silk, including $1.25 cost of box. How many pieces of silk were in the case, if he received $53.25 for each piece.

BILLS.

70. A **Bill** of Goods is a written statement of the goods sold, giving the quantity and price of each article and total cost, also the date of the sale, with the names of the buyer and seller.

71. The party who owes is called a *Debtor*, and the party to whom a debt is owed is called a *Creditor*.

SPECIMEN OF A BILL.

TORONTO, August 25, 1895.

James Brown, Esq.,

Bought of C. Meredith.

1895.				$	c.
Jan.	19	15	lb. Coffee at 32c	4	80
"	23	16	" Lard at 15c	2	40
Feb.	2	25	" Sugar at 13c	3	25
"	20	16	" Ham at 16c	2	56
				$13	01

NOTE. Pupils should be required to make out original bills. Having obtained reliable information from parents and others in reference to prices, trade customs, etc., they can make out bills, and furnish items for bills to be made out by the class.

SPECIMEN OF RECEIPTED BILL.

TORONTO, Sept. 1, 1895.

John Smith, Dr.

To George Brown.

1895.					$	c.
Jan. .	1	To 75 lb. of sugar at $0.12.	$9	00		
Feb. .	2	" 47 yds. of cloth " 3.25.	152	75		
					161	75
		Cr.				
Jan. .	7	By 75 bu. of corn at $0.78..	$58	50		
Feb. .	2	" 43 bu. of apples " 1.25..	53	75		
					112	25
		Balance due.....			$49	50

1895, Sept. 15th.

Received Payment,

George Brown.

Exercise xlii.

Make out bills for the following accounts, supplying dates:—

1. Mr. J. Jones bought of R. Walker 10 yards silk, at $2.50; 12 yards flannel, at 40 cents; 16 yards calico, at 15 cents.

2. Mr. Brown bought of McClung & Bros. 10 pounds tea, at 75 cents; 8 pounds raisins, at 18 cents; 5 pounds rice, at 10 cents; 12 pounds butter at 21 cents.

3. James Taylor bought of Thomas Yellowlees 5 quires foolscap, at 25 cents; 1 Hamblin Smith's Arithmetic, at 75 cents; 3 rolls wall paper, at 45 cents; 4 dolls, at 25 cents

4. David Montgomery bought of F. F. McArthur 20 yards cotton, at 11 cents; 15 yards print, at 16 cents; 12 yards braid, at 6 cents; 3 pair gloves, at 27 cents; 26 yards dress goods, at 63 cents; 1 hat, at $5.25.

5. Robert Davey bought of Murdoch Bros. 18 bags salt, at 75 cents; 4 barrels plaster, at 98 cents; 10 pounds coffee, at 35 cents; 1 chest tea, 18 pounds, at 65 cents; 48 grain bags, at $3.60 a doz.

6. Levi Van Camp sold Wm. Burns & Co. 257 bushels wheat at $1.12; 475 bushels oats, at 36 cents; 45 bushels corn, at 76 cents; 175 bushels pease, at 82 cents; 367 bushels barley, at 69 cents.

7. A. Thompson bought of A. Harrison 32 pounds sugar, at 12 cents; 11 pounds coffee, at 35 cents; 26 pounds soap, at 8 cents; 14 pounds rice, at 9 cents; 7 pounds fish, at 15 cents; 18 pounds crackers, at 12 cents.

8. W. West bought of T. Brown 27 pair calfskin boots, at $4.50; 96 pair gaiters, at $3.25; 126 pair overshoes, at 91 cents; 18 pair slippers, at 95 cents; 75 pair heavy boots, at $2.75.

9. Mrs. Jones bought of R. Walker & Co. 25 yards calico, at 12 cents; 12 spools cotton, at 5 cents; 16 yards alpaca, at 75 cents; 17 yards muslin at 18 cents; 6 skeins tape, at 2 cents.

10. Murdoch Bros. sold to A. Preston the following: 27 yards calico, at 13 cents; 45 yards muslin, at 18 cents; 16 yards linen, at 15 cents; 17 yards cambric, at 15 cents; and 9 handkerchiefs, at 45 cents; and took in exchange 12 bushels of potatoes, at 65 cents; 3 barrels apples, at $3.25; 13 pounds butter, at 35 cents, and the remainder in cash. How much cash was paid? Make out a receipted bill.

EXAMINATION PAPERS.

I.

1. A farmer gave $43.50 for sheep, at the rate of $7.25 for 3 sheep; how many did he buy?

No. of sheep bought for $7.25 = 3 sheep,

" " $1 = $\frac{3}{7.25}$ "

" " $43.50 = $\frac{43.50 \times 3}{7.25}$ sheep,

= 18 sheep.

2. If 18 chickens cost $4.20, how much will 3 chickens cost?

3. A merchant bought 9 pieces of cloth, each containing 50 yards, for which he paid $2317.50; what was the cost of a single yard?

4. A banker has $20000 in cash; he pays for 50 shares of stock, at $97.50 a share; and 100 shares, at $110 a share; how many shares, at $41.25 each, can he buy with the remainder of his money?

5. I owe $276 and paid $17.25 on it; how many times must I pay such a sum to cancel the debt?

II.

1. I retail envelopes at 12 cents a pack, gaining 3 cents on each pack of 24; what did they cost me per 1000?

Cost of 24 envelopes = 9 cents,
" 1 " = $\frac{9}{24}$ "
" 1000 " = $\frac{9000}{24}$ "
= $3.75.

2. A grocer sold 9760 pounds of flour, at $1.25 per 100 pounds; what was the amount of the sale?

3. Messrs. Smith & Co. burn in their store, in a year, 62560 cubic feet of gas; what is their gas bill for a year, at $4.50 per 1000 feet?

4. A man bought a quantity of coal for $250, and by retailing it at $5.75 a ton, he gained $37.50; how many tons did he buy?

5. The charge of sending a telegram to a certain place is 40 cents for ten words, and 5 cents for each additional word; what would a despatch of 24 words cost me?

III.

1. A horse worth $150, and 7 cows at $25 each, were exchanged for 57 sheep and $25.75 in money; what was the price of a sheep?

Value of horse and cows = $150 + 7 × $25 = $325.
Value of sheep = $325 − $25.75 = $299.25.
Hence " 57 sheep = $299.25;
therefore " 1 sheep = $\frac{299.25}{57}$
= $5.25.

2. A merchant bought 5 pieces of cloth of equal lengths, at $3.25 a yard: he gained $18.75 on the whole cost by selling four of the pieces for $750; how many yards were there in each piece?

3. At an election there were three candidates A, B, and C: the total number of votes polled was 7734. The successful candidate, A, got 203 votes more than C, who got 107 votes less than one-third of the total vote polled; what was A's majority over B?

4. A father divided his property worth $1767 among his three sons A, B, and C, in such a way that A got as much as B and C together, and B and C shared alike; what was C's share?

5. If the continued product of 275, 376, 481 and 196 be divided by $77 \times 28 \times 17 \times 55$ what will be the quotient?

IV.

1. A merchant expended $547.40 for cloth. He sold a certain number of yards for $522, at $1.45 per yard, and gained on what he sold $108. How many yards did he buy, and how much did he gain per yard on the cloth he sold?

2. A farmer exchanged 390 bushels of wheat worth $1.20 a bushel, for an equal number of bushels of barley at 75 cents a bushel, and oats at 12 cents a bushel; how many bushels of each did he receive?

3. John Turner has manufactured in 4 years 7740 pair of shoes, making each successive year 250 pair more than the year before, how many pair did he manufacture the first year?

4. If 80 men have sufficient provisions for 75 days, and 20 men go away, how long will they last the rest?

5. The product of 275 and 86 is 23650; how much must be taken from the product to give the product of 275 and 82; and to give the product of 270 and 86?

CHAPTER III.

MEASURES AND MULTIPLES.

Section I.—Prime Numbers, Prime Factors, etc.

72. In the series of numbers 1, 2, 3, 4, etc., a distinction may be observed of *odd* and *even* numbers.

An **Odd** number is one which cannot be divided into two equal whole numbers, as 1, 3, 5, etc.

An **Even** number is one which can be divided into two equal whole numbers, as 2, 4, 6, etc.

73. There is another, and a more **important division** of numbers into two classes, one class consisting of numbers, each of which is divisible only by 1 and a number equal to itself, as 2, 3, 5, etc.; and the other class consisting of numbers which admit of other divisors, as 4, 6, 8, etc. The numbers of the former class are called *prime numbers;* and those in the latter class *composite numbers.* (Art. 61.)

74. A **Prime Number** is one which can be exactly divided only by unity and a number equal to itself.

75. The **Prime Factors** of a number are the prime numbers, which when multiplied together will produce it; thus, 2, 2 and 3 are the prime factors of 12.

Oral Exercises.

1. What are the prime factors of 30?

The prime factors of 30 are 3, 2 and 5, since these are the only prime number which multiplied together will produce 30.

2. Name the prime numbers from 16 to 53; from 53 to 101.

3. What are the prime factors of 12? 16? 15? 18?

4. What are the prime factors of 21? 25? 27? 32? 33? 34?

5. What prime factor is found in both 6 and 9?

6. What prime factor is found in both 20 and 26?

7. What prime factor is common to 12 and 30? 21 and 28?

8. What prime factor is common to 35 and 50? 14 and 70? 33 and 99? 42 and 48? 26 and 39?

76. To resolve a number into its Prime Factors.

Ex. 1. Find the prime factors of 105.

$$\begin{array}{r} 3\,)\,105 \\ 5\,)\,35 \\ \hline 7 \end{array}$$

Dividing 105 by 3, a prime factor, we have 35; dividing 35 by 5, a prime factor, we have 7, a prime number, therefore the prime factors of 105 are 3, 5, 7.

Exercise xliii.

Find the prime factors of

1. 48.	5. 175.	9. 429.	13. 313.
2. 72.	6. 270.	10. 276.	14. 336.
3. 81.	7. 160.	11. 800.	15. 855.
4. 108.	8. 325.	12. 180.	16. 1155.

What prime factors are common to

17. 50 and 70?	19. 63 and 147?
18. 81 and 96?	20. 120 and 600?

Section II.—Cancellation.

77. Cancellation is the process of shortening operations in division by rejecting or *cancelling* equal factors common to both dividend and divisor.

Ex. 1. Divide 28 by 8.

$$\frac{28}{8} = \frac{4 \times 7}{4 \times 2} = \frac{7}{2} = 3\tfrac{1}{2}$$

Write the divisor, 8, under the dividend 28. Resolve 28 into 4×7, and 8 into 4×2. Cancelling the common factor 4 in dividend and divisor, we have 7 divided by 2 or $3\frac{1}{2}$.

The same result will be obtained by dividing both dividend and divisor by 4.

Hence, *Cancelling a common factor in both dividend and divisor does not change the quotient.*

Exercise xliv.

1. Divide $16 \times 4 \times 5$ by $8 \times 2 \times 10$.
2. Divide $7 \times 16 \times 6$ by $14 \times 3 \times 8$.
3. Divide $9 \times 7 \times 16 \times 16$ by $21 \times 32 \times 2$.
4. Divide $27 \times 12 \times 14$ by $9 \times 4 \times 7$.
5. Divide $72 \times 45 \times 140$ by $18 \times 24 \times 35$.
6. Divide $24 \times 32 \times 36 \times 144$ by $64 \times 108 \times 8$.
7. How many yards of muslin, worth 12 cents a yard, may be bought for 16 pounds of butter, worth 15 cents a pound?
8. How many bushels of potatoes at 75 cents a bushel must a farmer give for 36 yards of carpet worth $1.50 a yard?
9. A tailor bought 12 pieces of cloth, each containing 22 yards, worth $2.25 a yard; he made 27 suits of clothes; how much must he get per suit so as not to lose?
10. If a farmer exchange 25 bushels of wheat at $1.20 a bushel for cloth at 40 cents a yard, how many yards does he get?
11. Three pieces of cloth containing 30 yards each, worth $5 a yard, were exchanged for 5 pieces of cloth containing 45 yards each; what was the second kind worth per yard?
12. Divide the continued product of 16, 18, 24, 25, 36 and 45 by the continued product of 27, 72 and 100.

Section III.—The Highest Common Factor.

Oral Exercise.

Name a common factor

1. Of 6 and 9.
2. Of 12 and 10.
3. Of 27 and 24.
4. Of 16 and 20.
5. Of 12 and 18.
6. Of 16 and 40.

What is the highest common factor

7. Of 12 and 10?	10. Of 24 and 72?
8. Of 20 and 15?	11. Of 24 and 12?
9. Of 25 and 50?	12. Of 72 and 144?

78. A Common Factor of two or more numbers is a number that will exactly divide each of the given numbers.

79. The Highest Common Factor, called also the *Greatest Common Measure* or *Highest Common Divisor*, of two or more numbers is the largest number that will exactly divide each of the given numbers.

Ex. 1. Find the highest common factor of 18, 36, and 72.

6) 18, 36, 72
3) 3, 6, 12
1, 2, 4,

We place the numbers as in the margin. By dividing each number by 6, we take out the common factor 6; by dividing each of the quotients by 3 we take out the common factor 3: since the quotients, 1, 2, 4, have no factor common to all of them, 6 and 3 are all the common factors of the given numbers, hence 6×3, or 18 is their H. C. F.

Hence, *To find the H. C. F. of two or more numbers, we divide by any common factor of all the numbers; we then divide the quotients in the same manner, and thus continue until the quotients have no common factor; the product of all the divisors will be the highest common factor.*

Exercise xlv.

Find the H. C. F.

1. Of 15, 20, 30.	6. Of 84, 126, 210.
2. Of 16, 20, 24.	7. Of 120, 240, 72.
3. Of 24, 96, 80.	8. Of 44, 110, 77.
4. Of 28, 56, 42.	9. Of 75, 300, 450.
5. Of 30, 50, 60.	10. Of 144, 576, 720.

11. A man has two logs which he wishes to cut into boards of equal length; one is 24 feet, and the other is 16 feet long; what is the greatest length into which the boards can be cut?

12. What is the greatest equal length into which two trees can be cut, one being 105 feet in length and the other 84 feet?

13. Three pieces of carpet, of 48, 64 and 80 yards respectively, if cut into the longest possible equal lengths, will exactly cover a parlor floor, each piece being the length of the parlor; how long is the parlor?

14. A grocer has 136 quarts of strawberries, and 152 quarts of plums which he wishes to put into boxes, each box to hold the same number of quarts and the largest number possible; how many quarts may be put into each box?

15. If a pear costs an exact number of cents, what is the greatest number of pears you could buy with 180 cents or 225 cents, or 315 cents, so as to get the same number each time?

16. A certain school consists of 132 pupils in the lower school, and 99 in the upper school; how might each of these be divided so that the whole school should be distributed into equal sections?

80. To find the H. C. F. when the numbers are large.

Find H. C. F. or G. C. M. of 52 and 91.

$$
\begin{array}{r}
52\,)\,91\,(\,1 \\
\underline{52} \\
39\,)\,52\,(\,1 \\
\underline{39} \\
13\,)\,39\,(\,3 \\
\underline{39} \\
\end{array}
$$

This depends on the following principle:

Any number that will divide two other numbers will divide their sum or difference.

13 is a divisor of 39 and, therefore, of $13 + 39$ or 52. Since it divides 39 and 52 it also divides $39 + 52$ or 91; 13 is therefore a divisor or factor of 52 and 91.

It is also the Greatest Common Factor. If not, let a greater number divide 52 and 91, it will then divide 39, their difference, and dividing 39 and 52 it will also divide their difference, or 13. That is, a greater number than 13 will divide 13, which is impossible. 13 is therefore, the Greatest Common Divisor or H. C. F. of 52 and 91.

Hence, to find the H. C. F. of two numbers,

(1) *Divide the greater number by the lesser.*

(2) *Divide the lesser by the remainder.*

(3) *Divide the first remainder by the second, and continue this process, always dividing the last divisior by the last remainder. The last remainder which divides the preceding dirisor, is the Greatest Common Divisor or Highest Common Factor.*

To find the H. C. F. of more than two numbers, first find the H. C. F. of two of them; then find the H. C. F. of the common factor thus found and a third number; and so on through all the numbers. The last common factor found will be the H. C. F. of all the numbers.

Exercise xlvi.

Find the H. C. F. of

1. 115 and 161.
2. 333 and 592.
3. 697 and 820.
4. 392 and 672.
5. 405 and 900.
6. 1220 and 2013.
7. 6006 and 3318.
8. 2871 and 4213.
9. 43902 and 49593.
10. 23940 and 28350.
11. 1435, 1084 and 2135.
12. 14385, 20391 and 19287.

13. A grocer has two hogsheads of sugar, one containing 1104 pounds, and the other 1288 pounds. He wishes to put this sugar into barrels, each barrel to contain the same number of pounds, and this the greatest number possible; of how many pounds must each barrel consist?

14. *A* and *B* purchased horses at the same rate per head; the value of *A's* horses was $623; and of *B's* $1068; what was the number purchased by each?

15. A land-owner has three fields containing 24 acres, 18 acres and 42 acres. He wishes to cut them into smaller fields of an equal number of acres each; how large will the fields be?

16. Three men living on a new street own land fronting as follows: *A*, 600 feet; *B*, 720 feet; *C*, 900 feet. They wish to cut their land into lots of an equal width; how wide will the lots be, and how many will each have?

17. A man has a lot of the following dimensions: 450 feet long on the north side; 540 on the east side; 420 on the south side; and 495 on the west side. What is the greatest length of his panels of fence if they are equal, and how many rails and posts will be required if the fence is five rails high?

Section IV.—Least Common Multiple.

Oral Exercises.

1. What number is three times 5? four times 7?

A number which is one or more times another number, is called a *multiple* of that number.

2. What number is a multiple of 3? of 5? of 9?

3. Name two multiples of 8; three multiples of 7.

4. What number is a multiple of both 4 and 6? 3 and 5?

5. What multiple is common to both 3 and 4? 4 and 7?

6. Name all the multiples of 4 from 3 to 30.

7. What is the least number of which 3 and 5 are factors?

8. What is the least number exactly divisible by 3, 4, and 8?

9. What is the least number exactly divisible by 10 and 12? by 8 and 12? by 6 and 10? by 12 and 18?

10. James has just enough money to buy oranges at 5 cents each, pears at 4 cents each, or tops at 6 cents each; how much money has he?

81. A **Multiple** of a number is a number that is one or more times that number.

82. A **Common Multiple** of two or more numbers is a

number that is one or more times each of the given numbers. Thus 24 is a common multiple of 4 and 6, because it is 6 times 4 and 4 times 6.

83. The **Least Common Multiple** (L. C. M.) of two or more numbers is the least number that is one or more times each of them.

Ex. 1. Find the least common multiple of 24, 20, and 33.

$24 = 2 \times 2 \times 2 \times 3$

$20 = 2 \times 2 \times 5$

$33 = 3 \times 11$

L. C. M. $= 2 \times 2 \times 2 \times 3 \times 5 \times 11 = 1320$.

The L. C. M. of the given numbers must contain the factors 2, 2, 2 and 3 to be divisible by 24; it must contain the factors 2, 2, and 5, to be divisible by 20; it must contain the factors 3 and 11 to be divisible by 33. Since the number 1320 contains all these factors and no others, it is the least common multiple of 24, 20, and 33.

Hence, *To find the L. C. M. of two or more numbers we find the prime factors of the numbers, and take the product of these factors, using each the greatest number of times it occurs in any of the given numbers.*

84. When the several numbers are not large, the process may be shortened by successive divisions of the given numbers, by *prime factors*, which are common to two or more of the given numbers. By this means, all the divisors will consist of the prime factors common to two or more of the numbers, and the numbers left after the divisions will be the factors which are not common to any two of the numbers. Then the product of these common prime factors, and the factors which are not common, will be the least common multiple of the given numbers.

Ex. 2. Find the L. C. M. of 15, 21, 36, and 42.

$$\begin{array}{r|l} 2 & 15,\ 21,\ 36,\ 42 \\ \hline 2 & 15,\ 12,\ 18,\ 21 \\ 3 & 15,\ 6,\ 9,\ 21 \\ & 5,\ 2,\ 3,\ 7 \end{array}$$

Here 2, 2, 3 are the prime factors common to two or more of the numbers, and 5, 2, 3, 7 are the factors not common.

L. C. M. $= 2 \times 2 \times 3 \times 5 \times 2 \times 3 \times 7 = 2520$.

Exercise xlvii.

Find the L. C. M.

1. Of 15, 10 and 5.
2. Of 20, 10 and 30.
3. Of 9, 12 and 18.
4. Of 10, 25 and 30.
5. Of 24, 30 and 36.
6. Of 5, 9, 12 and 15.
7. Of 12, 15, 18 and 24.
8. Of 22, 55, 77 and 110.
9. Of 15, 30, 42 and 72.
10. Of 21, 54, 56 and 84.
11. Of 5, 7, 16, 28, 48 and 21.
12. Of 16, 12, 14, 32, 50 and 75.
13. Of 15, 18, 24, 40, 50, 60 and 90.
14. Of the even numbers from 14 to 28 inclusive.
15. Of the odd numbers from 13 to 25 inclusive.
16. What is the least number which divided by 8, by 12, and by 14 gives in each case the remainder 5?
17. What is the least sum of money for which I can purchase sheep at $6, or cows at $28, or horses at $150 a head?
18. What is the least number of bushels of wheat that would make an exact number of full loads for three drays hauling respectively 24, 30 or 36 bushels a load?
19. What is the least number of cents with which you could buy an exact number of lemons at 6 cents each; or oranges at 8 cents; or bananas at 10 cents; or pine apples at 16 cents?
20. How many bushels would fill a number of barrels, each containing 3 bushels, or a number of sacks, each containing 4 bushels, or a number of casks, each containing 14 bushels, the quantity to be the same in each case, and the smallest possible?
21. *A*, *B*, *C*, and *D* start together, and travel the same way round an island which is 600 miles in circuit. *A* goes 20 miles per day, *B* 30, *C* 25, and *D* 40. How long must their journeyings continue in order that they may all come together again?
22. A boy on counting his marbles 3 at a time, or 4 at a time, or 5 at a time, always has one over; but on counting them 7 at a time he has none over. What is the least number of marbles he can have?

EXAMINATION PAPERS.

I.

1. How do you determine whether a given number is prime or composite? Which of the following numbers are prime and which composite:—611, 643, 707, 757, 991, 1089?

2. Divide the continued product of 6, 15, 16, 21, 12, 21, and 17 by the continued product of 2, 10, 9, 8, 36, 7, and 51.

3. What is the least number of dollars that will purchase an exact number of cows at $24 each, sheep at $6 each, or horses at $127 each?

4. What is the least number which divided by 18, 21, and 30, gives 13 for remainder in each case?

5. A man owns 3 tracts of land containing 525, 725, and 875 acres respectively. He wishes to divide each tract into lots that will contain the same number of acres, and this the largest number possible; of how many acres must his lots consist?

II.

1. Define Highest Common Factor and Prime Factor, and explain when a number is Odd and when Even.

2. Find the largest number which will divide 941 and 1484, leaving as remainders 16 and 9.

3. What is the quotient of $144 \times 75 \times 15 \times 32 \times 23$ divided by $432 \times 25 \times 8 \times 30$?

4. What is the least number of marbles that can be divided equally among 16, 21, 24 or 30 boys?

5. A can dig 24 post holes in a day; B can dig 25, and C 30 in the same time. What is the smallest number which will furnish exact days' labor either for each working alone or for all working together?

III.

1. The product of four consecutive numbers is 73440; find the numbers?

2. What is the least number of acres in a farm that can be exactly divided into lots of 12 acres, 15 acres, 18 acres, or 25 acres each?

3. A farmer sold 4 loads of apples, each containing 15 barrels, and each barrel 3 bushels at 60 cents a bushel. He received as payment 6 barrels of pork, each weighing 200 pounds; what was the pork worth a pound?

4. The product of two numbers is 152368, and 7 times one of them is 2996; what is the other one?

5. How many rails will enclose a field 7163 feet long by 3315 feet wide, provided the fence is straight, 6 rails high, the rails of equal length, and the longest that can be used?

IV.

1. A farmer exchanged 9 tubs of butter, each containing 56 pounds, worth 25 cents per pound, for 4 chests of tea, each containing 42 pounds; what was the tea worth per pound?

2. What is the smallest sum of money with which I can buy sheep at $5 each, cows at $24 each, oxen at $54 each, or horses at $135 each?

3. Divide the continued product of 51, 72, 144, 972, and 750 by the continued product of 9, 17, 18, 24, 36, and 45.

4. Find the least number which divided by 1595, 2530, and 3168, will leave the same remainder, 719.

5. The following are the prime factors of a number: 2, 2, 3, 5, 5, 7, 11, 11, 13, 19, 89, and 227; find the number.

V.

1. State and prove the rule for finding the H. C. F. of two numbers, and find the H. C. F. of 1287000 and 504504.

2. Find the L. C. M. of 16, 24, and 30, and explain the method.

3. A school was found to contain such a number of boys, that when arranged in sixes, sevens, nines, or twelves, there were always five over; how many children, *at least*, did the school contain?

4. The fore and hind wheels of a carriage are 12 and 15 feet in circumference; find the least number of revolutions of each that will give the same length.

CHAPTER IV.

FRACTIONS.

Section I.—Definitions.

Oral Exercises.

1. If an apple is divided into two equal parts, what is one of these equal parts called?

2. How many halves are there in anything? Write down one-half.

3. When I divide an orange into three equal parts, what is one of these equal parts called? What are two of them called?

4. How many thirds are there in anything? How many fourths are there in anything?

5. How would you get fourths, fifths, sixths?

6. How many thirds make a whole? How many fourths? sevenths? tenths?

7. Into how many equal parts must a thing be divided to get halves? fifths? sevenths? eighths?

8. Two halves of an apple are equal to how many whole apples?

9. What are four fourths of a pear equal to?

10. Which are the *smaller*, halves or thirds? Halves or fourths? thirds or fourths?

The value of the part varies according to the number of equal parts into which the whole is divided. The more parts it is divided into, the smaller they must be.

One half a thing is greater than one third; one third is greater than one fourth.

85. A **Fraction** is one or more of the equal parts of a unit.

86. Fractions are divided into two classes, **Common, or Vulgar Fractions,** and **Decimal Fractions.**

87. A Common Fraction is one which is expressed by two numbers one placed above the other with a line between them; thus four-fifths is written $\frac{4}{5}$; nine-elevenths, $\frac{9}{11}$; ten thirty-fifths, $\frac{10}{35}$.

88. One of these equal parts is called the **Fractional Unit,** and instead of the name of this unit being written *after* the number of such units, as in whole numbers, it is placed *under* it. Thus, three apples is written 3 apples, and 3 fourths, $\frac{3}{4}$.

89. The number written below the line is called the **Denominator** or "*name-giver*," because it indicates the name of the fractional unit, *i. e.*, it shows into how many equal parts the whole is divided.

90. The number written above the line is called the **Numerator,** *i.e.*, the "*numberer*" or "*counter*," because it indicates how many of the parts named by the denominator are to be taken.

91. The Terms of a fraction are the numerator and the denominator. Thus, $\frac{5}{8}$ is a fraction—5 and 8 are its *terms*.

92. A Proper Fraction is one the numerator of which is *less* than its denominator. Thus, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{7}{8}$ are proper fractions.

93. If we cut an apple into *two* equal parts, one-half will be represented by $\frac{1}{2}$.

If we cut an apple into *four* equal parts one-half will be represented by $\frac{2}{4}$.

If we cut an apple into *eight* equal parts, one-half will be represented by $\frac{4}{8}$.

$$\therefore \frac{1}{2} = \frac{2}{4} = \frac{4}{8}.$$

Similarly,

If we cut an apple into *three* equal parts, one-third will be represented by $\frac{1}{3}$.

$\therefore$ Stands for the word *therefore*.

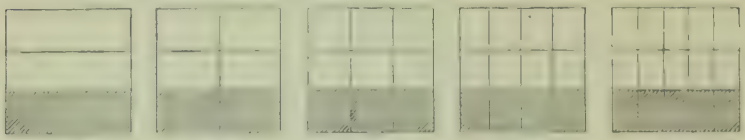
If we cut an apple into *nine* equal parts, one-third will be represented by $\frac{3}{9}$.

If we cut an apple into *eighteen* equal parts, one-third will be represented by $\frac{6}{18}$.

$$\therefore \frac{1}{3} = \frac{3}{9} = \frac{6}{18}.$$

Hence we conclude that, **The value of a fraction is not altered by multiplying or dividing both its numerator and its denominator by the same number.**

This is the most important proposition in fractions. It may be illustrated by colored paper cut in squares of about four inches, such as are used for folding in the kindergarten. Each pupil has several pieces of paper, does the folding, and draws the conclusions, the teacher simply directing and questioning.



$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15}.$$

This illustration is so simple that it requires no explanation.

All the proposition in fractions may be illustrated by paper folding.

94. The usual definition of a fraction is given in Art. 85, but by the help of the following proposition, which is best explained by an example, we shall be able to obtain another definition of a fraction, which is sometimes useful.

Ex. 1. Prove that $\frac{2}{5}$ of $1 = \frac{1}{5}$ of 2.

Since $1 =$ *five*-fifths of a unit,

$2 =$ *ten*-fifths of a unit;

$\therefore \frac{1}{5}$ of $2 = \frac{1}{5}$ of *ten*-fifths of a unit

$=$ *two*-fifths of a unit

$= \frac{2}{5}$ of 1;

$\therefore \frac{1}{5}$ of $2 = \frac{2}{5}$ of 1.

Hence, *We may define a fraction as a simple manner of indicating that its numerator is to be divided by its denominator.*

95. Since 3 apples multiplied by 2 = 6 apples,
so 3 eighths ($\frac{3}{8}$) " 2 = 6 eighths ($\frac{6}{8}$);
$\therefore 2 \times \frac{3}{8} = \frac{6}{8}$.

Hence, *To multiply a fraction by a whole number, we simply multiply the numerator by the whole number, and retain the denominator.*

Since 8 marbles divided by 2 = 4 marbles,
so 8 ninths ($\frac{8}{9}$) " 2 = 4 ninths ($\frac{4}{9}$),
$\therefore \frac{8}{9} \div 2 = \frac{4}{9}$.

Hence, *A fraction is divided by a whole number by dividing the numerator by the number and retaining the denominator.*

96. From the preceding article it appears that fractions may, in general, be treated as whole numbers. Indeed, they differ from whole numbers simply in the unit employed. Thus, in 20 feet, 1 foot is the unit or base of the collection and in $\frac{4}{9}$, the fractional unit is $\frac{1}{9}$, four of them being taken or collected to form the fraction.

The fractional unit is always equal to 1 divided by the denominator.

Section II.—Reduction of Fractions.

Case I.

97. To reduce whole or mixed numbers to improper Fractions.

98. A **Mixed Number** consists of a whole number and a fraction; as $3\frac{1}{5}$, $4\frac{3}{7}$, etc.

99. An **Improper Fraction** is one whose numerator is not less than its denominator.

Oral Exercises.

1. How many halves in 5 apples?
   In 1 apple there are 2 halves, and in 5 apples there are 5 times 2 halves, or 10 halves.

2. How many halves in 6? In 10? In 13? In 40?
3. How many fourths in 4? In 6? In 9? In 12?
4. How many fifths in $4\frac{3}{5}$?

In 1 there are 5 *fifths*, and in 4 there are 4 times 5 fifths or 20 fifths, which added to 3 fifths, make 23 fifths; therefore $4\frac{3}{5}=\frac{23}{5}$.

5. How many fourths in $7\frac{3}{4}$? In $5\frac{2}{4}$? In $9\frac{1}{4}$? In $12\frac{3}{4}$?
6. How many sixths in $4\frac{2}{6}$? In $8\frac{3}{6}$? In $9\frac{1}{6}$? In $11\frac{5}{6}$?

Ex. 1. Reduce $27\frac{3}{4}$ to fourths.

$$27\tfrac{3}{4}=27+\tfrac{3}{4};$$

Now, $1=\frac{4}{4}$;

$$\therefore \quad 27=\frac{27\times 4}{4}=\frac{108}{4},$$

$$\therefore \quad 27+\tfrac{3}{4}=\frac{108}{4}+\frac{3}{4}=\frac{111}{4}=\frac{27\times 4+3}{4}.$$

Hence, *To reduce mixed numbers to improper fractions, we multiply the whole number by the denominator of the fractions, add the numerator to the product, and write the denominator under the sum.*

Exercise xlviii.

Reduce to improper fractions

1. $3\frac{4}{7}$.	7. $35\frac{11}{12}$.	13. $237\frac{13}{152}$.
2. $4\frac{1}{5}$.	8. $82\frac{15}{17}$.	14. $304\frac{75}{286}$.
3. $9\frac{2}{7}$.	9. $51\frac{17}{23}$.	15. $1306\frac{9}{178}$.
4. $4\frac{5}{9}$.	10. $86\frac{5}{19}$.	16. $2500\frac{184}{973}$.
5. $11\frac{1}{12}$.	11. $99\frac{1}{11}$.	17. $1001\frac{11}{101}$.
6. $12\frac{3}{8}$.	12. $78\frac{19}{99}$.	18. $2897\frac{716}{999}$.

Case II.

100. To reduce improper fractions to whole or mixed numbers.

Ex. 2. Reduce $\frac{87}{11}$ to a mixed number.

Since dividing both terms of a fraction by the same number does not change its value (Art. 93), we have,

$$\frac{87}{11}=\frac{87\div 11}{11\div 11}=\frac{7\frac{10}{11}}{1}=7\tfrac{10}{11}.$$

Hence, *To reduce an improper fraction to a whole or mixed number we simply divide the numerator by the denominator.*

Exercise xlix.

Reduce the following fractions to whole or mixed numbers:

1. $\frac{19}{5}$.
2. $\frac{31}{6}$.
3. $\frac{45}{7}$.
4. $\frac{75}{10}$.
5. $\frac{99}{3}$.
6. $\frac{102}{6}$.
7. $\frac{196}{16}$.
8. $\frac{982}{15}$.
9. $\frac{725}{45}$.
10. $\frac{325}{25}$.
11. $\frac{476}{17}$.
12. $\frac{982}{19}$.
13. $\frac{4407}{136}$.
14. $\frac{5919}{59}$.
15. $\frac{17007}{33}$.
16. $\frac{32460}{48}$.
17. $\frac{25650}{694}$.
18. $\frac{72648}{139}$.

Case III.

101. To reduce a fraction to its lowest terms.

102. A Fraction is in its **Lowest Terms** when the numerator and denominator have no common factor.

Ex. 3. Reduce $\frac{48}{108}$ to its lowest terms.

$$\frac{48}{108} = \frac{12}{27} = \frac{4}{9}.$$

Dividing both terms of $\frac{48}{108}$ by the common factor 4, reduces it to $\frac{12}{27}$; dividing both terms of this fraction by 3, reduces it to $\frac{4}{9}$. Since $\frac{4}{9}$ has its numerator and denominator prime to each other it is in its *lowest terms.*

We might have found the H. C. F. of the numerator and denominator and divided both terms by it at once.

Hence, *To reduce a fraction to its lowest terms we divide both terms by a common factor, and the result again by a common factor, and so on till the terms have no common factor.*

Or, *we may divide both terms of the fraction by their Highest Common Factor.*

Exercise l.

Reduce the following fractions to their lowest terms:

1. $\frac{15}{20}$.
2. $\frac{18}{27}$.
3. $\frac{14}{35}$.
4. $\frac{24}{36}$.
5. $\frac{75}{90}$.
6. $\frac{80}{112}$.
7. $\frac{210}{294}$.
8. $\frac{648}{720}$.
9. $\frac{288}{864}$.
10. $\frac{792}{864}$.
11. $\frac{840}{4312}$.
12. $\frac{1176}{1512}$.
13. $\frac{6161}{7171}$.
14. $\frac{5400}{7200}$.
15. $\frac{1296}{2584}$.
16. $\frac{1896}{2502}$.
17. $\frac{304}{1072}$.
18. $\frac{660}{1155}$.
19. $\frac{672}{1056}$.
20. $\frac{1584}{5940}$.
21. $\frac{4235}{7315}$.
22. $\frac{3276}{4914}$.
23. $\frac{2109}{2553}$.
24. $\frac{2418}{2597}$.

Case IV.

103. To reduce a Compound Fraction to a Simple one.

104. A **Compound Fraction** is a fraction of a fraction; as $\frac{1}{2}$ of $\frac{3}{4}$; $\frac{3}{7}$ of $\frac{4}{5}$, etc.

105. A **Simple Fraction** is one in which both numerator and denominator are whole numbers; as $\frac{3}{4}$, $\frac{5}{6}$, etc.

Oral Exercises.

1. What is $\frac{1}{2}$ of 6 apples? of 10 boys? of 16 cents?

2. What is $\frac{1}{2}$ of 6 ninths? of 10 elevenths? of 16 twentieths?

3. What is $\frac{1}{2}$ of $\frac{6}{9}$?

Since $\frac{1}{2}$ of 6 apples = 3 apples,
$\frac{1}{2}$ of 6 ninths $(\frac{6}{9})$ = 3 ninths $(\frac{3}{9})$.

4. What is $\frac{1}{3}$ of $\frac{12}{17}$? of $\frac{15}{22}$? of $\frac{18}{25}$? of $\frac{9}{22}$?

5. What is $\frac{1}{5}$ of $\frac{2}{7}$? $\frac{3}{5}$ of $\frac{2}{7}$? $\frac{3}{5}$ of $\frac{1}{16}$? $\frac{1}{5}$ of $\frac{30}{21}$?

6. What is $\frac{1}{3}$ of $\frac{1}{4}$?

$$\frac{1}{4} = \frac{3}{12},$$
$\frac{1}{3}$ of $\frac{1}{4}$ = $\frac{1}{3}$ of $\frac{3}{12} = \frac{1}{12}$.

7. What is $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{5}$ of $\frac{1}{2}$? $\frac{1}{4}$ of $\frac{1}{3}$? $\frac{1}{4}$ of $\frac{1}{5}$?

8. A boy had $\frac{1}{5}$ of a dollar, and lost $\frac{1}{4}$ of it? what part of a dollar did he lose?

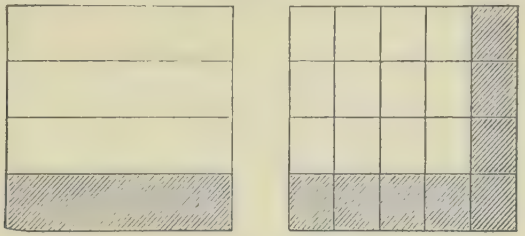
9. A man owned $\frac{1}{4}$ of a farm, and sold $\frac{1}{6}$ of his share: how much did he sell?

10. I had $\frac{1}{5}$ a ton of coal, and gave my neighbour $\frac{1}{3}$ of it: he gave his brother $\frac{1}{8}$ of his share: how much did his brother get?

Ex. 4. Reduce $\frac{3}{5}$ of $\frac{7}{8}$ to a simple fraction.

$$\frac{7}{8} = \frac{35}{40},$$
$\frac{1}{5}$ of $\frac{7}{8}$ = $\frac{1}{5}$ of $\frac{35}{40} = \frac{7}{40}$;
$\therefore \frac{3}{5}$ of $\frac{7}{8} = 3 \times \frac{7}{40} = \frac{21}{40} = \frac{3 \times 7}{5 \times 8}$
$= \frac{\text{the product of the numerators}}{\text{the product of the denominators}}$

This may be illustrated by paper folding; thus:



$\frac{4}{5}$ of $\frac{3}{4}$ = $\frac{12}{20} = \frac{3}{5}$.

Hence, *To reduce a compound fraction to a simple one, multiply the numerators together for a new numerator, and the denominators together for a new denominator.*

NOTE.—Before performing the multiplication, mixed numbers should be reduced to improper fractions, and any factor common to a numerator and denominator cancelled.

Exercise li.

Simplify the following fractions:

1. $\frac{4}{7}$ of $\frac{5}{11}$.
2. $\frac{2}{3}$ of $\frac{9}{22}$.
3. $\frac{3}{4}$ of $4\frac{1}{4}$.
4. $2\frac{3}{4}$ of $\frac{5}{9}$.
5. $\frac{5}{7}$ of $\frac{4}{5}$ of $\frac{2}{3}$.
6. $\frac{4}{9}$ of $\frac{3}{11}$ of $\frac{5}{8}$.
7. $\frac{3}{11}$ of $\frac{5}{7}$ of $2\frac{4}{9}$.
8. $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$.
9. $\frac{4}{7}$ of $\frac{3}{4}$ of $\frac{9}{10}$ of $\frac{7}{15}$.
10. $\frac{5}{8}$ of $\frac{6}{7}$ of $\frac{7}{9}$ of $\frac{8}{11}$.
11. $\frac{5}{9}$ of $\frac{2}{5}$ of $\frac{9}{12}$ of $\frac{2}{3}$.
12. $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{1}{28}$ of 7.
13. $\frac{3}{5}$ of $\frac{4}{11}$ of $\frac{17}{23}$ of $5\frac{3}{4}$.
14. $\frac{22}{35}$ of $\frac{8}{11}$ of $\frac{15}{22}$ of $9\frac{5}{8}$.
15. $\frac{5}{7}$ of $8\frac{3}{4}$ of $\frac{1}{5}$ of $2\frac{1}{7}$.
16. $\frac{5}{16}$ of $\frac{8}{9}$ of $\frac{1}{5}$ of $4\frac{1}{4}$.
17. $\frac{2}{9}$ of $\frac{5}{3}$ of $\frac{3}{8}$ of 9.
18. $\frac{1}{9}$ of $\frac{8}{15}$ of $3\frac{15}{16}$ of 6.

Exercise lii.

1. Some boys owned $\frac{2}{3}$ of a boat; they sold $\frac{3}{4}$ of their share; what part of the boat did they sell?

2. Having $\frac{3}{4}$ of a bushel of potatoes I gave away $\frac{5}{6}$ of what I had; what part of a bushel did I give away?

3. A boy had $\frac{9}{10}$ of a dollar, and spent $\frac{2}{5}$ of it; how much did he spend?

4. A gentleman owning $\frac{3}{5}$ of a factory gave $\frac{2}{9}$ of what he owned to his son; what part of the whole factory was the son's share?

5. A has $\frac{4}{7}$ of a ton of hay, which is $\frac{3}{4}$ as much as B has; how much has B?

$\frac{3}{4}$ of what B has $= \frac{4}{7}$ of a ton;
$\therefore \frac{1}{4}$ " " $= \frac{1}{3}$ of $\frac{4}{7}$ of a ton $= \frac{4}{21}$ of a ton;
$\therefore \frac{4}{4}$ " " $= 4 \times \frac{1}{21}$ of a ton $= \frac{4}{21}$ of a ton;
$\therefore$ B has $\frac{4}{21}$ of a ton.

6. A owns $\frac{4}{7}$ of a railroad, and $\frac{3}{4}$ of this is $3\frac{3}{7}$ times what B owns; how much does B own?

7. How many acres of land has B if $\frac{3}{10}$ of 18 is $\frac{6}{25}$ of his number?

8. A's money equals $\frac{5}{12}$ of \$8750, and A's is $\frac{3}{60}$ of B's money; how much money has B?

Case V.

106. To reduce Fractions to equivalent ones having the least common denominator.

1. Let the denominators be prime to each other and the fractions in their simplest form.

Ex. 5. Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ to equivalent fractions with *least* common denominator.

Since multiplying the terms of a fraction by the same number does not change its value, we have,

$$\frac{2}{3} = \frac{4 \times 5 \times 2}{4 \times 5 \times 3} = \frac{40}{60},$$

$$\frac{3}{4} = \frac{3 \times 5 \times 3}{3 \times 5 \times 4} = \frac{45}{60},$$

and $$\frac{4}{5} = \frac{3 \times 4 \times 4}{3 \times 4 \times 5} = \frac{48}{60}.$$

Hence, *To reduce fractions to equivalent ones having a common denominator, we multiply both terms of each fraction by all the denominators except its own.*

Exercise liii.

Reduce to equivalent fractions having a common denominator

1. $\frac{3}{5}$, $\frac{7}{9}$.
2. $\frac{6}{7}$, $\frac{9}{10}$.
3. $\frac{8}{9}$, $\frac{7}{11}$.
4. $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{7}$.
5. $\frac{5}{7}$, $\frac{2}{3}$, $\frac{4}{5}$.
6. $\frac{3}{5}$, $\frac{2}{3}$, $\frac{3}{4}$.
7. $\frac{3}{5}$, $\frac{1}{3}$, $\frac{2}{7}$, $\frac{1}{4}$.
8. $\frac{5}{7}$, $\frac{3}{5}$, $\frac{4}{9}$, $\frac{11}{13}$.
9. $\frac{3}{5}$, $\frac{5}{8}$, $\frac{4}{9}$, $\frac{8}{11}$.

2. Let the denominators be not prime to each other.

Ex. 6. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$ to equivalent fractions with least common denominator.

L. C. M. of 3, 4, 6, 8 = 24.

$\frac{2}{3} = \frac{8 \times 2}{8 \times 3} = \frac{16}{24}$

$\frac{3}{4} = \frac{6 \times 3}{6 \times 4} = \frac{18}{24}$

$\frac{5}{6} = \frac{4 \times 5}{4 \times 6} = \frac{20}{24}$

$\frac{7}{8} = \frac{3 \times 7}{3 \times 8} = \frac{21}{24}$

We find the L. C. M. of the denominators to be 24, hence 24 is the least common denominator. Dividing 24 by 3, the denominator of $\frac{2}{3}$, we find we must multiply 3 by 8 to produce 24, similarly with the other fractions.

Hence, *To reduce fractions to equivalent ones having a common denominator, when the denominators are not prime to each other, we find the least common multiple of the denominators, divide this by each denominator, and multiply both terms of the fraction by the quotient.*

Exercise liv.

Reduce the following to equivalent fractions with the least common denominator:

1. $\frac{2}{3}$, $\frac{5}{6}$, $\frac{3}{5}$.
2. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$.
3. $\frac{7}{8}$, $\frac{9}{10}$, $\frac{7}{20}$.
4. $\frac{5}{9}$, $\frac{7}{12}$, $\frac{11}{18}$.
5. $\frac{5}{6}$, $\frac{4}{9}$, $\frac{7}{12}$,
6. $\frac{3}{4}$, $\frac{7}{10}$, $\frac{5}{6}$.
7. $\frac{1}{4}$, $\frac{7}{12}$, $\frac{11}{20}$.
8. $\frac{5}{6}$, $\frac{4}{5}$, $\frac{2}{3}$.
9. $\frac{7}{8}$, $\frac{3}{4}$, $\frac{4}{5}$.
10. 2, $\frac{3}{10}$, $\frac{4}{5}$.
11. $2\frac{1}{2}$, $3\frac{1}{3}$, $\frac{5}{6}$.
12. $\frac{7}{9}$, $4\frac{1}{3}$, 5.
13. 3, $4\frac{1}{2}$, $\frac{1}{16}$.
14. $\frac{7}{8}$, $\frac{5}{14}$, $\frac{5}{12}$, $\frac{13}{42}$.
15. $\frac{4}{7}$, $\frac{5}{11}$, $\frac{9}{77}$, 6.
16. $\frac{4}{15}$, $4\frac{1}{3}$, $\frac{4}{7}$, 2.
17. $\frac{7}{12}$, $\frac{17}{24}$, $\frac{11}{36}$, $\frac{55}{64}$.
18. $2\frac{1}{4}$, $7\frac{1}{5}$, $3\frac{2}{9}$, $4\frac{7}{12}$.

107. Comparison of Fractions with respect to Magnitude.

To compare fractions we must express them in terms of the same fractional unit, that is, we must bring them to a common denominator. When they are so expressed they are compared as other numbers.

Ex. 7. Arrange the fractions $\frac{5}{6}$, $\frac{4}{9}$, $\frac{7}{12}$, in order of magnitude.

Reducing to equivalent fractions having a common denominator we have $\frac{30}{36}$, $\frac{16}{36}$, $\frac{21}{36}$; hence the order of magnitude is $\frac{4}{9}$, $\frac{7}{12}$, $\frac{5}{6}$.

If two fractions happen to have the same numerator, that which has the smaller denominator is the greater; for its units are greater, and there are the same number of them in each.

Exercise lv.

Find which is the greater,

1. $\frac{4}{7}$ or $\frac{5}{9}$.
2. $\frac{8}{9}$ or $\frac{25}{27}$.
3. $\frac{7}{15}$ or $\frac{10}{21}$.
4. $\frac{7}{15}$ or $\frac{11}{18}$.
5. $\frac{17}{20}$ or $\frac{21}{25}$.
6. $\frac{15}{19}$ or $\frac{17}{21}$.
7. $\frac{5}{34}$ or $\frac{7}{51}$.
8. $\frac{7}{40}$ or $\frac{11}{70}$.
9. $\frac{9}{11}$ or $\frac{13}{15}$.

Which is the greatest and which is the least of the following:

10. $\frac{7}{15}$, $\frac{10}{20}$, $\frac{16}{35}$?
11. $\frac{5}{11}$, $\frac{9}{22}$, $\frac{16}{33}$?
12. $\frac{3}{16}$, $\frac{2}{9}$, $\frac{7}{32}$?
13. $\frac{9}{10}$, $\frac{17}{20}$, $\frac{21}{25}$?
14. $\frac{9}{11}$, $\frac{13}{15}$, $\frac{17}{21}$?
15. $\frac{5}{6}$, $\frac{6}{7}$, $\frac{8}{9}$?

Arrange in ascending order of magnitude

16. $\frac{17}{81}$, $\frac{3}{18}$, $\frac{5}{27}$, $\frac{4}{84}$, $\frac{2}{9}$.
17. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{13}{20}$.

Section III.—Addition.

Case I.

108. To add proper Fractions.

Oral Exercises.

1. What is the sum of 2 apples, 3 apples, and 5 apples?

2. What is the sum of 2 elevenths, 3 elevenths, and 5 elevenths?

3. How many ninths are $\frac{2}{9}$, $\frac{1}{9}$, $\frac{1}{9}$, and $\frac{3}{9}$?

4. James paid $\$\frac{1}{8}$ for a slate, $\$\frac{3}{8}$ for a reader, and $\$\frac{1}{8}$ for an arithmetic; how much did he pay for all?

5. Mary paid $\$\frac{1}{8}$ for some ribbon, and $\$\frac{3}{8}$ for a pair of gloves; how much did she pay for both?

6. Jane bought $\frac{3}{4}$ of a yard of ribbon at one time and $\frac{7}{8}$ of a yard at another time; how much did she buy at both times?

$\frac{3}{4} = \frac{6}{8}$, and $\frac{6}{8} + \frac{7}{8} = \frac{13}{8} = 1\frac{5}{8}$. She, therefore, bought $1\frac{5}{8}$ yards.

7. A farmer sold $\frac{1}{5}$ of his grain to one man, and $\frac{1}{6}$ of it to another; how much did he sell altogether?

$\frac{1}{5} = \frac{6}{30}$, and $\frac{1}{6} = \frac{5}{30}$; $\therefore \frac{1}{5} + \frac{1}{6} = \frac{6}{30} + \frac{5}{30} = \frac{11}{30}$.

8. If I pay $\frac{1}{4}$ of a dollar for butter, $\frac{3}{5}$ of a dollar for eggs, and $\frac{1}{2}$ a dollar for cheese; how much do I pay for all?

9. What is the sum of $\frac{1}{4}$ and $\frac{1}{5}$? of $\frac{1}{6}$ and $\frac{1}{4}$? of $\frac{1}{4}$ and $\frac{3}{8}$?

10. What is the sum of $\frac{1}{2}$ and $\frac{1}{3}$? of $\frac{1}{2}$ and $\frac{2}{5}$, of $\frac{2}{3}$ and $\frac{3}{4}$?

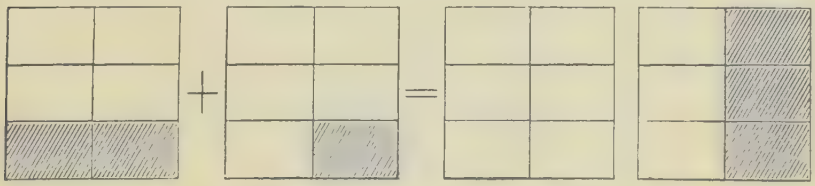
Ex. 1. Find the sum of $\frac{2}{5}$, $\frac{5}{6}$, and $\frac{3}{10}$.

$$\frac{2}{5} + \frac{5}{6} + \frac{3}{10} = \frac{12}{30} + \frac{25}{30} + \frac{9}{30} = \frac{46}{30} = \frac{23}{15} = 1\frac{8}{15}.$$

In this example we are required to add fifths, sixths, and tenths together. As the addends have not the same name we cannot add them until they are changed into others having the same fractional unit. We, therefore, change the fractions into others having a common denominator, we then add the numerators together for a new numerator, and call the sum 30ths. We reduce the improper fraction to its lowest terms, and then to the mixed number, $1\frac{8}{15}$.

This may be illustrated by paper folding as follows:

Add $\frac{2}{3}$ and $\frac{5}{6}$. $\frac{2}{3} = \frac{4}{6}$.



$\frac{4}{6}$ + $\frac{5}{6}$ = $\frac{9}{6}$ = $1\frac{3}{6}$ = $1\frac{1}{2}$.

Hence, *To add fractions, we reduce them to equivalent ones having a common denominator; we then add the numerators together for a new numerator and place the sum over the common denominator.*

Exercise lvi.

Add together the following fractions:

1. $\frac{2}{3}$ and $\frac{3}{4}$.
2. $\frac{4}{5}$ and $\frac{7}{8}$.
3. $\frac{5}{12}$ and $\frac{7}{18}$.
4. $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{3}{4}$.
5. $\frac{1}{2}$, $\frac{3}{5}$ and $\frac{7}{8}$.
6. $\frac{4}{25}$, $\frac{9}{40}$ and $\frac{3}{20}$.
7. $\frac{5}{6}$, $\frac{2}{7}$, $\frac{3}{4}$ and $\frac{5}{12}$.
8. $\frac{3}{4}$, $\frac{7}{16}$, $\frac{5}{6}$ and $\frac{5}{12}$.
9. $\frac{2}{5}$, $\frac{4}{7}$, $\frac{13}{21}$ and $\frac{7}{15}$.
10. $\frac{1}{4}$, $\frac{1}{5}$, $\frac{5}{6}$ and $\frac{2}{9}$.
11. $\frac{2}{3}$, $\frac{4}{5}$, $\frac{11}{12}$ and $\frac{29}{42}$.
12. $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{12}$ and $\frac{1}{17}$.

Case II.

109. To add mixed numbers.

Ex. 2. Add together $2\frac{1}{4}$, $3\frac{1}{5}$, $7\frac{5}{12}$ and $\frac{7}{10}$.

$$2\tfrac{1}{4} + 3\tfrac{1}{5} + 7\tfrac{5}{12} + \tfrac{7}{10}$$
$$= 2 + 3 + 7 + \tfrac{1}{4} + \tfrac{1}{5} + \tfrac{5}{12} + \tfrac{7}{10}.$$
$$= 12 + \tfrac{15}{60} + \tfrac{12}{60} + \tfrac{25}{60} + \tfrac{42}{60}$$
$$= 12 + \tfrac{130}{60} = 12 + 2\tfrac{1}{6} = 14\tfrac{1}{6}.$$

NOTE.—When there are mixed numbers in the example we add the sum of the whole numbers to the sum of the fractions.

Exercise lvii.

Find the sum of the following fractions:

1. $2\frac{1}{2}$, $3\frac{2}{3}$ and $4\frac{1}{5}$.
2. $2\frac{1}{3}$, $\frac{5}{12}$ and $7\frac{1}{16}$.
3. $2\frac{5}{7}$, $3\frac{1}{3}$ and $4\frac{2}{11}$.
4. $3\frac{2}{3}$, $1\frac{3}{4}$ and $\frac{1}{15}$.
5. $1\frac{5}{19}$, $\frac{11}{38}$ and $2\frac{5}{7}$.
6. $30\frac{1}{5}$, $4\frac{9}{10}$ and $10\frac{5}{8}$.
7. $2\frac{1}{3}$, $4\frac{1}{5}$, $7\frac{1}{12}$ and $8\frac{5}{15}$.
8. $7\frac{3}{5}$, $10\frac{1}{15}$, $4\frac{1}{3}$ and $7\frac{7}{30}$.
9. $1\frac{1}{8}$, $3\frac{1}{3}$, $2\frac{3}{4}$ and $5\frac{5}{12}$.
10. $4\frac{1}{3}$, $5\frac{1}{4}$, $3\frac{1}{5}$, and $9\frac{1}{15}$.
11. $4\frac{2}{5}$, $3\frac{1}{3}$, $4\frac{5}{9}$ and $\frac{4}{15}$.
12. $5\frac{1}{3}$, $6\frac{3}{4}$, $3\frac{5}{9}$ and $1\frac{1}{12}$.

Section IV. -Subtraction.

Case I.

110. To subtract one Fraction from another.

Oral Exercises.

1. John has 7 marbles. James has 4; how many marbles has John more than James?

2. John has $\frac{7}{12}$ of an apple, James has $\frac{4}{12}$ of an apple; how much has John more than James?

3. How much less is $\frac{3}{8}$ than $\frac{7}{8}$? $\frac{3}{8}$ than $\frac{5}{8}$? $\frac{3}{8}$ than $\frac{8}{8}$?

4. John has $\frac{1}{3}$ of an apple, James has $\frac{1}{4}$ of an apple; how much has John more than James?

$\frac{1}{3} = \frac{4}{12}$, and $\frac{1}{4} = \frac{3}{12}$; $\therefore \frac{1}{3} - \frac{1}{4} = \frac{4}{12} - \frac{3}{12} = \frac{1}{12}$.

5. A boy spent $\frac{1}{2}$ of his money for a coat and $\frac{1}{4}$ of it for a hat; how much had he left?

6. What is the difference between $\frac{1}{3}$ and $\frac{1}{2}$? $\frac{2}{3}$ and $\frac{3}{4}$?

7. What is the difference between $\frac{2}{3}$ and $\frac{5}{12}$? between $\frac{3}{4}$ and $\frac{1}{8}$?

Ex. 1. From $\frac{7}{12}$ take $\frac{7}{20}$.

$$\frac{7}{12} - \frac{7}{20} = \frac{35}{60} - \frac{21}{60} = \frac{14}{60} = \frac{7}{30}.$$

In this example we are required to take twentieths from twelfths. As we can only subtract numbers that have the same name, we must change the fractions into equivalent ones, having a common denominator. $\frac{7}{12}$ becomes $\frac{35}{60}$, and $\frac{7}{20}$ becomes $\frac{21}{60}$. We now find the difference between $\frac{35}{60}$ and $\frac{21}{60}$ to be $\frac{14}{60}$, which reduced to its lowest terms is $\frac{7}{30}$.

Illustrate, as in addition, thus:—

Subtract $\frac{3}{4}$ from $\frac{7}{8}$. $\frac{3}{4} = \frac{6}{8}$.

$\frac{7}{8} - \frac{6}{8} = \frac{1}{8}$.

Hence, *To subtract one fraction from another we reduce the fractions to equivalent ones, having a common denominator; we then subtract the numerator of the subtrahend from the numerator of the minuend; and place the difference over the common denominator.*

Exercise lviii.

Find the difference between,

1. $\frac{5}{7}$ and $\frac{3}{5}$.
2. $\frac{7}{9}$ and $\frac{2}{5}$.
3. $\frac{5}{9}$ and $1\frac{1}{2}$.
4. $\frac{17}{20}$ and $\frac{4}{5}$.
5. $2\frac{2}{3}$ and $4\frac{3}{8}$.
6. $\frac{2}{7}$ and $\frac{4}{15}$.
7. $\frac{5}{6}$ and $\frac{19}{20}$.
8. $\frac{5}{7}$ and $\frac{13}{100}$.
9. $\frac{2}{19}$ and $\frac{5}{9}$.
10. $\frac{9}{14}$ and $\frac{16}{63}$.
11. $\frac{14}{39}$ and $\frac{19}{65}$.
12. $\frac{9}{35}$ and $\frac{7}{30}$.

Case II.

111. To subtract when one or both the fractions are mixed numbers.

Ex. 2. From $16\frac{1}{3}$ take $9\frac{3}{4}$.

$16\frac{1}{3} = 16\frac{4}{12}$; $16\frac{4}{12} + \frac{12}{12} = 16\frac{16}{12}$.

$9\frac{3}{4} = 9\frac{9}{12}$; $9\frac{9}{12} + 1 = 10\frac{9}{12}$,

$6\frac{7}{12}$.

We first reduce the fractional parts of the given numbers to their least common denominator, 12. Then, since $\frac{9}{12}$ cannot be taken from $\frac{4}{12}$, we add 1 or $\frac{12}{12}$ to both minuend and subtrahend (Art. 33). This gives $16\frac{16}{12}$ for minuend and $10\frac{9}{12}$ for subtrahend. Then subtracting the fractions and integers separately, we have $16\frac{16}{12} - 10\frac{9}{12} = 6\frac{7}{12}$, the required result.

NOTE. The fractional part of the result can be obtained much more readily by subtracting the numerator of the fraction in the subtrahend from the common denominator and adding the remainder to the numerator of the fraction in the minuend. Thus $12 - 9 = 3$; $3 + 4 = 7$. This is the numerator of the fractional part of the result. The integral part is obtained as above.

Or we proceed as follows:

Add to $\frac{3}{4}$ such a fraction as will make it a whole number, and add the same fraction to the minuend, thus: (Art. 33)

$$\left.\begin{matrix} 16\frac{1}{3} + \frac{1}{4} \\ 9\frac{3}{4} + \frac{1}{4} \end{matrix}\right\} - \left\{\begin{matrix} 16\frac{7}{12} \\ 10 \end{matrix}\right.$$

$6\frac{7}{12}$.

Exercise lix.

Find the value of

1. $3\frac{3}{7} - 1\frac{4}{5}$.
2. $2\frac{1}{9} - 1\frac{1}{10}$.
3. $6\frac{1}{2} - 2\frac{1}{12}$.
4. $8\frac{5}{7} - 5\frac{1}{9}$.
5. $3\frac{5}{12} - 1\frac{5}{6}$.
6. $2\frac{5}{6} - 1\frac{7}{12}$.
7. $5\frac{3}{14} - 2\frac{20}{21}$.
8. $1\frac{1}{8} - 1\frac{3}{11}$.
9. $9 - 4\frac{3}{7}$.
10. $18\frac{2}{9} - 3\frac{5}{6}$.
11. $28\frac{16}{63} - 3\frac{9}{14}$.
12. $25\frac{7}{10} - 14\frac{13}{15}$.

Exercise lx.

Practical Problems.

1. The sum of two numbers is $26\frac{1}{4}$ and the less is $7\frac{5}{7}$; what is the greater?

2. From a barrel of vinegar containing $31\frac{1}{2}$ gallons, $14\frac{13}{16}$ gallons were drawn; how much was there left?

3. To what fraction must the sum of $\frac{1}{3}$ and $\frac{1}{15}$ be added that the sum may be $\frac{14}{15}$?

4. From a piece of silk containing $35\frac{2}{5}$ yards, $14\frac{2}{3}$ yards were sold; how much remained in the piece?

5. From \$10, \$2$\frac{1}{2}$ were given to James, \$3$\frac{1}{4}$ to Jane, \$1$\frac{1}{2}$ to Emily, and the remainder to Mary; what did she receive?

6. *A* has two farms, one of $70\frac{3}{7}$ acres, and the other of $118\frac{19}{35}$ acres; if he sells $87\frac{19}{21}$ acres, how much land has he left?

7. How much paper has a printer left if he had on hand $30\frac{7}{12}$ reams, and has used $7\frac{2}{5}$ reams for one job, and $8\frac{3}{4}$ reams for another?

8. A grocer, having mixed $15\frac{3}{4}$ pounds of tea with $32\frac{11}{21}$ pounds of a different kind, sold all the mixture but $13\frac{1}{3}$ pounds, how much did he sell?

9. *B* started on a journey of 100 miles; the first day he travelled $30\frac{5}{12}$ miles; the second day $36\frac{5}{9}$ miles; how far has he yet to go?

10. Henry had \$47$\frac{1}{3}$, and James as much lacking \$9$\frac{4}{15}$; how many dollars had James?

11. The selling price of a horse was \$125$\frac{3}{7}$; the gain was \$26$\frac{3}{5}$; what was the cost price?

12. Find the sum of the greatest and least of the fractions, $\frac{3}{8}$, $\frac{5}{21}$, $\frac{4}{9}$, $\frac{7}{20}$, the sum of the other two, and the difference of these sums.

Exercise lxi.

Practical Problems.

1. \$249$\frac{1}{4}$ is \$134$\frac{1}{3}$ less than the value of my horse and carriage; what are they worth?

2. A boy paid $\$\frac{1}{5}$ for a ball, $\$\frac{1}{6}$ for a slate, $\$\frac{7}{8}$ for a knife and $\$1\frac{7}{12}$ for a book; how much did he spend?

3. What is the entire weight of 4 crocks of butter weighing as follows:—The first, $10\frac{1}{4}$ pounds; the second, $11\frac{3}{5}$ pounds; the third, $13\frac{5}{16}$ pounds; and the fourth, $14\frac{1}{2}$ pounds?

4. Three men bought a horse; the first paid $\$41\frac{5}{8}$, the second paid $\$53\frac{1}{12}$, and the third as much as the other two; what was the cost of the horse?

5. A grocer has three barrels of molasses; the first contains $27\frac{3}{5}$ gallons, the second, $42\frac{7}{16}$, and the third, $36\frac{5}{6}$ gallons; how many gallons are in the three barrels?

6. What number is that from which if $5\frac{1}{12}$ is taken, the remainder will be $2\frac{17}{18}$?

7. A merchant sold $34\frac{5}{9}$ yards of cloth for $\$94\frac{5}{16}$, $39\frac{5}{27}$ yards for $\$124\frac{5}{6}$, and $70\frac{3}{4}$ yards for $\$184\frac{3}{8}$; how many yards of cloth did he sell and how much did he receive for the whole?

8. Four geese weigh respectively $9\frac{5}{8}$, $10\frac{2}{3}$, $12\frac{7}{16}$ and $11\frac{5}{6}$ pounds; what is their entire weight?

9. A lady hired a gardener at 15 cents an hour for 3 days; how much did she pay him if he worked $6\frac{5}{12}$ hours the first day, $7\frac{3}{8}$ the second, and $5\frac{3}{4}$ the third?

10. If $5\frac{2}{7}$ gallons of brandy are mixed with $1\frac{9}{12}$ gallons of water and $3\frac{5}{21}$ gallons of whiskey, how many gallons are there in the mixture?

11. *A* paid $\$46\frac{1}{2}$ for an ox, and $\$57\frac{5}{12}$ more than this for a horse; for how much must he sell them to gain $\$26\frac{3}{4}$?

12. *A* owns $71\frac{5}{8}$ acres of land, *B* owns $112\frac{5}{16}$ acres, *C* owns $217\frac{1}{2}\frac{7}{4}$ acres, and *D* owns $372\frac{1}{4}\frac{3}{2}$ acres, how many acres do they together own?

Section V. Multiplication and Division of Fractions.

Case I.

112. To multiply a fraction by a whole number.

Oral Exercises.

1. If 5 cents are multiplied by 3, what is the product?

2. If 3 marbles are multipled by 5, what is the product?

3. If 3 sevenths are multiplied by 5, what is the product? 3 sevenths ($\frac{3}{7}$) multiplied by 5 = 15 sevenths ($\frac{15}{7} = 2\frac{1}{7}$).

4. How much will 5 pair of ducks cost at $\$\frac{3}{4}$ a pair?

5. How much will 10 yards of cloth cost at $\$\frac{1}{4}$ a yard?

6. If it requires $\frac{3}{4}$ of a yard of cloth to make a vest, how many yards will it require to make 8 vests?

7. If a man earns $\$\frac{4}{5}$ in 1 day, how much will he earn at the same rate in 10 days?

8. If a hat cost $\$\frac{5}{6}$ how much will 12 hats cost?

9. If 1 yard of muslin cost $\$\frac{5}{8}$, what would be the cost of 4 yards?

10. If a man can plough $\frac{8}{9}$ of an acre of land in 1 day, how many acres could he plough in 7 days?

11. If a barrel of flour cost $\$8\frac{3}{4}$, what will 5 barrels cost?

Multiply the fractional and integral parts separately, and add the products.

12. How much is 6 times $\frac{7}{8}$? $\frac{9}{10}$? $\frac{11}{121}$? $\frac{3}{6}$?

13. How much is 8 times $\frac{10}{12}$? $\frac{9}{12}$? $\frac{5}{8}$? $\frac{5}{6}$?

14. How much is 10 times $1\frac{3}{20}$? $2\frac{3}{4}$? $6\frac{1}{5}$? $3\frac{5}{12}$?

Ex. 1. Multiply $\frac{3}{10}$ by 5.

Since 3 apples multiplied by 5 = 15 apples,
so 3 tenths ($\frac{3}{10}$) multiplied by 5 = 15 tenths ($\frac{15}{10}$);

$$\therefore 5 \times \tfrac{3}{10} = \tfrac{15}{10} = \tfrac{5 \times 3}{10};$$

but $\frac{5 \times 3}{10} = \frac{3}{2}$(Art. 77) $= \frac{3}{10 \div 5}$.

Hence, *To multiply a fraction by any number we either multiply the numerator by the number or divide the denominator by it.*

Exercise lxii.

Multiply

1. $\frac{15}{18}$ by 9.
2. $\frac{12}{15}$ by 8.
3. $\frac{9}{28}$ by 7.
4. $\frac{14}{15}$ by 10.
5. $\frac{16}{21}$ by 49.
6. $\frac{11}{14}$ by 91.
7. $\frac{15}{12}$ by 21.
8. $\frac{8}{19}$ by 24.
9. $\frac{15}{112}$ by 36

10. At $\$1\frac{1}{4}$ a day, how much does a man earn in 4 weeks of 6 days each?

11. What is the cost of 36 dozen eggs, at $36\frac{1}{2}$ cents a dozen?

12. At $\$16\frac{3}{4}$ a month what will a boy earn in 12 months?

13. What is the cost of 12 pounds of beef at $14\frac{3}{4}$ cents a pound?

14. What is the cost of 14 bushels of oats at $62\frac{1}{2}$ cents a bushel?

Case II.

113. To divide a fraction by a whole number.

Oral Exercises.

1. If 8 apples are divided by 4, what is the quotient?

2. If 8 ninths are divided by 4, what is the quotient?

3. Divide $\frac{8}{9}$ by 2; $\frac{6}{11}$ by 3; $\frac{16}{17}$ by 8.

4. If 3 ducks cost $\frac{6}{10}$ of a dollar, how much will 1 duck cost?

Cost of 3 ducks $= \$\frac{6}{10}$;

" 1 duck $= \frac{1}{3}$ of $\$\frac{6}{10} = \$\frac{2}{10} = \$\frac{1}{5} = 20$ cents.

5. If 3 caps cost $\frac{3}{10}$ of a dollar, how much will 1 cap cost?

6. William had $\frac{3}{4}$ of an orange, and divided it equally among 3 of his schoolmates; what part of an orange did he give to each?

7. A man shares $\frac{8}{9}$ of a ton of coal among 5 persons: how much does each get?

Share of 5 persons $= \frac{8}{9}$ of a ton;

" 1 person $= \frac{1}{5}$ of $\frac{8}{9}$ of a ton

$= \frac{8}{45}$ of a ton.

8. If 8 men can do $\frac{4}{5}$ of a piece of work, how much can 1 man do in the same time?

9. If 3 men together own $\frac{3}{4}$ of a vessel, what part of the vessel does 1 man own, if their shares are equal?

10. A lady gave $\frac{2}{3}$ of a pound of candy to her 4 sons and 2 daughters; what was the share of each?

Ex. 2. Divide $\frac{8}{9}$ by 4.

Since 8 apples divided by 4 = 2 apples,
so 8 ninths ($\frac{8}{9}$) divided by 4 = 2 ninths ($\frac{2}{9}$);

$$\therefore \frac{8}{9} \div 4 = \frac{2}{9} = \frac{8 \div 4}{9}.$$

We may obtain the same result by multiplying the denominator of $\frac{8}{9}$ by 4, and reducing the resulting fraction to its lowest terms; thus,

$$\frac{8}{9} \div 4 = \frac{8}{4 \times 9} = \frac{2}{9}.$$

Hence, *To divide a fraction by any number we either divide the numerator by the number or multiply the denominator by it.*

Exercise lxiii.

NOTE.—Reduce mixed numbers to improper fractions

Divide

1. $\frac{15}{17}$ by 5.	4. $\frac{7}{8}$ by 17.	7. $4\frac{2}{7}$ by 10.
2. $\frac{35}{40}$ by 7.	5. $7\frac{1}{5}$ by 6.	8. $129\frac{3}{5}$ by 16.
3. $\frac{63}{64}$ by 9.	6. $4\frac{2}{3}$ by 7.	9. $287\frac{5}{8}$ by 12.

10. If a man can reap $22\frac{3}{4}$ acres of wheat in 7 days, how much could he reap in 1 day?

11. If a man can cut $15\frac{3}{4}$ cords of wood in 7 days, how many cords could he cut in 1 day?

12. If a man can walk $38\frac{1}{3}$ miles in 10 hours, how far could he walk in one hour?

13. If 7 tons of coal cost $\$60\frac{2}{3}$, what is the price per ton?

Case III.

114. To multiply a whole number or a fraction by a fraction.

Oral Exercises.

1. If a yard of muslin cost 12 cents, how much will $\frac{1}{3}$ of a yard cost?

2. If a man earns $60 a month, how much will he earn in $\frac{1}{6}$ of a month?

3. If a ton of hay cost $25, how much will $\frac{4}{5}$ of a ton cost?

Cost of 1 ton = $25;

" $\frac{1}{5}$ " = $\frac{1}{5}$ of $25 = $5;

" $\frac{4}{5}$ " = $4 \times \$5 = \$20 = \$\frac{4 \times 25}{5}$.

4. If a house cost $800 and a barn $\frac{3}{4}$ as much, how much does the barn cost?

5. If $\frac{4}{5}$ of $50 is 8 times the cost of a shawl, what does the shawl cost?

6. How much will $\frac{3}{4}$ of a bushel of potatoes cost at $\$\frac{4}{5}$ a bushel?

Cost of 1 bushel $= \$\frac{4}{5}$;
" $\frac{1}{4}$ " $= \frac{1}{4}$ of $\frac{4}{5} = \$\frac{1}{5}$.
" $\frac{3}{4}$ " $= 3 \times \$\frac{1}{5} = \$\frac{3}{5}$.

7. John had $\$\frac{5}{6}$, and lost $\frac{2}{3}$ of it; what part of a dollar did he lose?

8. Robert had $\frac{3}{4}$ of a melon, and gave his brother $\frac{8}{9}$ of it; what part of the melon did he give away?

9. A man owning $\frac{3}{10}$ of a mill, sold $\frac{1}{3}$ of $\frac{4}{5}$ of what he owned; what part of the mill did he sell?

10. Thomas had $\frac{3}{8}$ of an orange, and gave to John $\frac{3}{5}$ of $\frac{2}{3}$ of what he had; what part of the whole orange did he give away?

Ex. 3. Multiply $\frac{4}{19}$ by $\frac{3}{7}$.

Here, $3 \times \frac{4}{19} = \frac{12}{19}$. (Art. 112.)

This result is evidently 7 times too great, because $\frac{4}{19}$ is not to be multiplied by 3 but by $\frac{1}{7}$ of 3 (Art. 94); we must therefore divide $\frac{12}{19}$ by 7. Hence we have

$$\frac{3}{7} \times \frac{4}{19} = \frac{12}{19} \div 7 = \frac{12}{19 \times 7} = \frac{3 \times 4}{7 \times 19}$$

(Art. 113.)

$$= \frac{\text{the product of the numerators}}{\text{the product of the denominators}}$$

Hence, *The product of two fractions is found by multiplying the two numerators together for the numerator, and the two denominators together for the denominator, of the product.*

In a similar way it may be shown that the product of more fractions than two is found by multiplying all the numerators together for the numerator, and all the denominators together for the denominator, of the product.

NOTE. Whole numbers may be regarded as fractions with unity for denominators.

Exercise lxiv.

NOTE.—Cancel the factors common to the numerators and the denominators.

Find the value of

1. $\frac{5}{6} \times 18$.
2. $\frac{8}{9} \times 45$.
3. $\frac{5}{6} \times 43$.
4. $\frac{7}{16} \times 124$.
5. $\frac{12}{15} \times \frac{9}{10}$.
6. $\frac{9}{10} \times \frac{5}{21} \times \frac{7}{25}$.
7. $\frac{6}{7} \times \frac{9}{12} \times \frac{14}{15}$.
8. $\frac{10}{11} \times \frac{3}{10} \times \frac{7}{12}$.
9. $\frac{45}{49} \times \frac{56}{65} \times \frac{91}{72}$.
10. $\frac{16}{21} \times \frac{9}{28} \times \frac{35}{54}$.
11. $\frac{9}{10} \times \frac{16}{27} \times \frac{35}{48}$.
12. $\frac{312}{728} \times \frac{20}{21} \times \frac{7}{9}$.

13. What should be paid for $\frac{1}{2}$ of $\frac{7}{8}$ of a pound of tea, at the rate of $\frac{13}{16}$ of a dollar per pound?

14. What should be paid for $\frac{2}{3}$ of a barrel of apples, if the whole barrel is worth $\frac{15}{16}$ of a dollar?

15. A has $\frac{3}{4}$ of \$375, B has $\frac{4}{5}$ as much, and C $\frac{3}{5}$ as much as both; how many dollars has each, and how many have they all?

Ex. 4. Multiply $6\frac{2}{3}$ by $7\frac{4}{5}$.

$7\frac{4}{5} = \frac{39}{5}$, and $6\frac{2}{3} = \frac{20}{3}$;

$\therefore 7\frac{4}{5} \times 6\frac{2}{3} = \frac{39}{5} \times \frac{20}{3} = \frac{13}{1} \times \frac{4}{1} = 52$

Exercise lxv.

Find the value of

1. $3\frac{1}{4} \times 5\frac{2}{7}$.
2. $6\frac{2}{3} \times 7\frac{2}{5}$.
3. $17\frac{2}{5} \times 16\frac{2}{3}$.
4. $39\frac{3}{5} \times 33\frac{1}{3}$.
5. $6\frac{2}{7} \times 4\frac{3}{11} \times 77 \times 4\frac{1}{4}$.
6. $3 \times 7\frac{1}{2} \times \frac{11}{15} \times 3\frac{8}{11}$.

7. If a cord of wood costs $\$4\frac{1}{4}$, what will $3\frac{1}{2}$ cords cost?

Cost of 1 cord $= \$4\frac{1}{4}$;

" $3\frac{1}{2}$ cords $= 3\frac{1}{2} \times \$4\frac{1}{4}$

$= \frac{7}{2} \times \$\frac{17}{4} = \$\frac{119}{8} = \$14\frac{7}{8}$.

8. If a pound of sugar is worth $9\frac{1}{2}$ cents, what will $4\frac{1}{2}$ pounds cost?

9. If a man reaps $3\frac{3}{4}$ acres of wheat in a day, how many could he reap in $2\frac{2}{3}$ days?

10. What would be the cost of $18\frac{3}{4}$ acres of land at $\$18\frac{3}{4}$ per acre?

11. If a ton of coal costs $\$6\frac{7}{8}$, what will be the cost of $9\frac{4}{5}$ tons at the same rate?

12. Mr. Jones rented a house at $\$42\frac{3}{4}$ a month, taking a lease for 5 years, but disposed of the lease at the end of $3\frac{1}{4}$ years; how much rent did he pay?

13. A bill of books at retail amounts to $\$375\frac{5}{8}$, but I got a reduction of $\frac{1}{3}$ for wholesale and $\frac{3}{50}$ for cash; what was the exact amount of the bill?

Case IV.

115. To divide a whole number or a fraction by a fraction.

Oral Exercises.

1. How many parcels of sugar, each containing 3 pounds, can you make out of 24 pounds?

2. How often is 3 pounds contained in 21 pounds?

3. If 24 apples are divided equally among 6 boys, how many apples will each boy receive?

4. How often is 6 apples contained in 24 apples?

5. When both *dividend* and *divisor* are concrete numbers, what kind of a number is the quotient? Give examples.

6. How often is 1 ninths ($\frac{1}{9}$) contained in 8 ninths ($\frac{8}{9}$)?

7. How often is 2 fifth ($\frac{2}{5}$) contained in 12 fifteenths ($\frac{12}{15}$)?

Reduce the fractions to equivalent ones having a common denominator.

8. How often is $\frac{2}{10}$ contained in $\frac{4}{5}$?

9. At $\frac{2}{10}$ of a dollar each, how many caps can I buy for $\$\frac{4}{5}$?

10. If a pound of coffee costs $\$\frac{1}{4}$, how much can be bought for $\$1\frac{1}{4}$?

11. At $\$\frac{3}{5}$ a yard, how many yards of cloth can be bought for \$6?

Yards bought for $\$\frac{3}{5} = 1$ yd. $= \frac{5}{5}$ yd.;
" " $\$1 = \frac{5}{5}$ yds. $\div \frac{3}{5} = \frac{5}{3}$ yards;
" " $\$6 = 6 \times \frac{5}{3}$ yards $= 10$ yards.

12. When apples are worth $\$\frac{3}{4}$ a bushel, how many bushels can be bought for $\$\frac{3}{5}$?

13. At $\$\frac{7}{8}$ a yard, how many yards of silk can be bought for \$4?

Ex. 5. Divide $\frac{2}{3}$ by $\frac{3}{5}$.

$\frac{2}{3} = \frac{10}{15}$, and $\frac{3}{5} = \frac{9}{15}$.

Since 10 apples divided by 9 apples $= \frac{10}{9}$,

so 10 fifteenths $(\frac{10}{15})$ divided by 9 fifteenth $(\frac{9}{15}) = \frac{10}{9}$,

$$\therefore \frac{2}{3} \div \frac{3}{5} = \frac{10}{15} \div \frac{9}{15} = \frac{10}{9}$$
$$= \frac{2 \times 5}{3 \times 3} = \frac{2}{3} \times \frac{5}{3}$$

=dividend multiplied by divisor inverted.

Or we may reason as follows:

Here, $\frac{2}{3} \div 3 = \frac{2}{9}$. (Art. 113,)

This result is evidently 5 times too small, because $\frac{2}{3}$ is not to be divided by 3, but by $\frac{1}{5}$ of 3. (Art. 94.) The true quotient must therefore be 5 times $\frac{2}{9}$. Hence

$\frac{2}{3} \div \frac{3}{5} = 5 \times \frac{2}{9} = \frac{5 \times 2}{3 \times 3} = \frac{5}{3} \times \frac{2}{3}$, as before.

Ex. 6. Divide 10 by $\frac{3}{4}$.

$10 = \frac{10}{1} = \frac{40}{4}$.

$10 \div \frac{3}{4} = \frac{40}{4} \div \frac{3}{4} = \frac{40}{3} = \frac{10 \times 4}{1 \times 3} = \frac{10}{1} \times \frac{4}{3}$

=dividend multiplied by divisor inverted.

Hence, *To divide one fraction by another, invert the divisor and multiply the dividend by the fraction thus formed.*

Exercise lxvi.

Divide

1. 10 by $\frac{5}{7}$.
2. 18 by $\frac{6}{7}$.
3. 30 by $\frac{7}{8}$.
4. 40 by $3\frac{1}{2}$.
5. $\frac{3}{4}$ by $\frac{3}{12}$.
6. $\frac{15}{17}$ by $\frac{9}{10}$.
7. $\frac{13}{14}$ by $\frac{7}{15}$.
8. $\frac{185}{286}$ by $\frac{15}{22}$.
9. $9\frac{3}{4}$ by $\frac{9}{10}$.
10. $7\frac{5}{24}$ by $12\frac{8}{15}$.
11. $21\frac{3}{7}$ by $12\frac{8}{21}$.
12. $45\frac{3}{5}$ by $2\frac{6}{9}$.

Exercise lxvii.

Practical Problems.

1. If $\frac{3}{4}$ of a yard of cloth cost 24 cents, what would a whole yard cost?

Cost of $\frac{3}{4}$ of a yard = 24 cents;

" $\frac{1}{4}$ " = 8 "

" $\frac{4}{4}$ " = 4×8 cents = 32 cents.

2. At $\$1\frac{7}{9}$ per bushel, how many bushels of wheat can be bought for $\$42\frac{2}{3}$?

3. If a ton of coal is worth $\$6\frac{2}{5}$, how many tons can be bought for $\$89\frac{6}{10}$?

4. If a bushel of apples cost $\$2\frac{1}{4}$, how many bushels could be bought for $\$60\frac{3}{4}$?

5. If a man earns $\$7\frac{1}{8}$ in a week, how long will it require him to earn $\$20\frac{3}{4}$?

6. A man divided $30\frac{1}{2}$ pounds of flour among the poor, giving to each $2\frac{1}{2}$ pounds; how many persons were there?

7. If $21\frac{3}{4}$ pounds of tea cost $\$18\frac{1}{2}\frac{9}{0}$, what will 1 pound cost?

8. If an errand boy earns $\$7\frac{1}{2}$ in a week, how long will it require him to earn $\$20\frac{1}{4}$?

9. A man raised $93\frac{3}{4}$ bushels of wheat on $8\frac{1}{3}$ acres of land; how many bushels per acre was that?

10. In how many days will a horse eat $329\frac{1}{3}$ pecks of oats, if he eats $1\frac{8}{11}$ pecks daily?

11. If $\frac{1}{8}$ of an acre of land sells for $\$30$, what will an acre sell for at the same rate?

12. The product of two numbers is 27, and one of them is $2\frac{2}{3}$; what is the other?

Section VI. Complex Fractions.

116. To Reduce a Complex Fraction to a Simple One.

117. A **Complex Fraction** is one in which either the numerator or denominator, or both, are fractions; as

$$\frac{\frac{3}{4}}{7}, \quad \frac{4}{2\frac{1}{2}}, \quad \frac{3\frac{1}{4}}{\frac{1}{8}}.$$

Ex. 1. Reduce $\dfrac{\frac{1}{9}}{\frac{3}{5}}$ to a simple fraction.

Since the numerator of a fraction is the dividend, and the denominator the divisor (Art. 91), we have simply to

divide the numerator, $\frac{1}{9}$, by the denominator, $\frac{3}{5}$, as in *division of fractions.*

hence, $\frac{\frac{1}{9}}{\frac{3}{5}} = \frac{1}{9} \div \frac{3}{5} = \frac{1}{9} \times \frac{5}{3} = \frac{5}{27}$.

Ex. 2. Reduce $\frac{8}{4\frac{1}{5}}$ to a simple fraction.

$$\frac{8}{1} \div 4\tfrac{1}{5} = \frac{8}{1} \times \frac{5}{21} = \frac{40}{21} = 1\tfrac{19}{21}.$$

In many cases it is simpler to multiply the numerator and denominator of the complex fraction by the L. C. M. of the denominators; thus,

$$\frac{8}{4\frac{1}{5}} = \frac{5 \times 8}{5 \times 4\frac{1}{5}} = \frac{40}{21} = 1\tfrac{19}{21}.$$

Ex. 3. Simplify $\frac{1+\frac{1}{3}}{1-\frac{1}{3}} \div \frac{1+\frac{3}{5}}{1-\frac{3}{5}}$.

Multiplying the numerator and denominator of the dividend by 3, and the numerator and denominator of the divisor by 5, we have

$$\frac{3+1}{3-1} \div \frac{5+3}{5-3} = \frac{4}{2} \div \frac{8}{2} = \frac{4}{8} = \frac{1}{2}.$$

Exercise lxviii.

1. $\frac{52}{3\frac{1}{4}}$.

2. $\frac{3\frac{3}{4}}{5}$.

3. $\frac{1\frac{4}{5}}{1\frac{1}{2}\frac{7}{5}}$.

4. $\frac{1\frac{1}{2}}{7\frac{17}{18}}$.

5. $\frac{4\frac{2}{9}}{2\frac{3}{8}}$.

6. $\frac{9\frac{7}{9}}{2\frac{1}{2}\frac{1}{7}}$.

7. $\frac{5\frac{3}{11}}{2\frac{7}{11}}$.

8. $\frac{8\frac{3}{4}}{5\frac{5}{8}}$.

9. $\frac{15\frac{3}{5}}{7\frac{4}{5}}$.

10. $\frac{9}{3\frac{3}{11}}$.

11. $\frac{23}{2\frac{2}{3}+\frac{2}{5}}$.

12. $\frac{2\frac{1}{2}+1\frac{2}{3}}{3\frac{2}{3}-2\frac{1}{2}}$.

13. $\frac{14\frac{1}{4}-6\frac{1}{2}}{3\frac{1}{3}+7\frac{1}{5}}$.

14. $\frac{4\frac{1}{12}+6\frac{1}{8}}{9\frac{1}{5}-3\frac{1}{4}}$.

15. $\frac{2\frac{1}{3}+1\frac{35}{48}}{9\frac{2}{7}-\frac{39}{112}}$.

16. $\frac{3\frac{1}{3} \text{ of } 1\frac{1}{5}}{1\frac{1}{3} \text{ of } 1\frac{1}{4}}$.

17. $\frac{3\frac{3}{7} \text{ of } 2\frac{11}{12}}{\frac{1}{33} \text{ of } 8\frac{9}{14}}$.

18. $\frac{4\frac{4}{15} \text{ of } 2\frac{5}{8}}{5\frac{1}{5} - 4\frac{1}{2}}$.

19. $\frac{2\frac{1}{4}+3\frac{1}{5}}{5\frac{1}{2} \times 1\frac{1}{3}}$.

20. $\frac{2\frac{1}{3}-1\frac{1}{5}}{1\frac{3}{5} \text{ of } 1\frac{7}{8}}$.

118. Brackets, which are of several kinds—*e.g.* ( ), { }, [ ],—are used to denote that all numbers included within

any pair of them are to be considered as forming but one number, and are therefore to be equally affected by any number not included within the same pair of brackets, thus

$$(6+3)\times 8 = 9\times 8 = 72.$$

Also, $[8+2\times\{9+3\times(4+3)+17\}+21]\times 9$
$= [8+2\times\{9+21+17\}+21]\times 9$
$= [8+94+21]\times 9$
$= 123\times 9$
$= 1107.$

119. A Vinculum is a sign sometimes used instead of brackets. It consists of a *line* drawn over the numbers to be considered as forming one number—thus,

$$2\times\overline{8+3} = 2\times 11 = 22.$$

In removing brackets from an expression, it is best to commence with the *innermost* and to remove the brackets one by one, the outermost last of all.

120. In finding the value of an involved fraction, it must be remembered that the signs $\times$, $\div$, and "of" *connect* the terms between which they are found into one quantity, while the signs $+$ and $-$ *separate* the terms between which they occur. *Brackets, however, should always be used where there is a possibility of ambiguity.*

The following cases will illustrate the generally received usage in Arithmetic respecting these signs:—

(1) *The operations indicated by "of," $\times$, and $\div$ should be performed before adding or subtracting.*

Ex. 4. $\frac{3}{4}+\frac{2}{3}$ of $\frac{9}{11}-\frac{1}{4}\div\frac{1}{3}+\frac{2}{3}\times\frac{6}{11}$
$=\frac{3}{4}+(\frac{2}{3}$ of $\frac{9}{11})-(\frac{1}{4}\div\frac{1}{3})+(\frac{2}{3}\times\frac{6}{11})$
$=\frac{3}{4}+\frac{6}{11}-\frac{3}{4}+\frac{4}{11}$
$=\frac{10}{11}.$

(2) *The operations indicated by $\times$ and $\div$ should be performed in the order in which they occur.*

Ex. 5. $\frac{2}{3}\times\frac{6}{11}\div\frac{3}{4}$
$=\frac{2}{3}\times\frac{6}{11}\times\frac{4}{3}$
$=\frac{16}{33}.$

Ex. 6. $\frac{2}{3}\div\frac{6}{11}\times\frac{3}{4}$
$=\frac{2}{3}\times\frac{11}{6}\times\frac{3}{4}$
$=\frac{11}{12}.$

Ex. 7. $\frac{3}{4} \times \frac{5}{6} \div \frac{2}{3} \times \frac{1}{4}$
$= \frac{3}{4} \times \frac{5}{6} \times \frac{3}{2} \times \frac{1}{4}$
$= \frac{15}{64}$.

(3) *The operation indicated by "of," should be performed before that indicated by* $\div$; this is the only case in which custom makes a distinction between $\times$ and "of."

Ex. 8. $\frac{3}{8}$ of $2\frac{5}{7} \div 1\frac{6}{7}$ of $\frac{3}{4}$
$= (\frac{3}{8} \times \frac{19}{7}) \div (\frac{13}{7} \times \frac{3}{4})$
$= \frac{3}{8} \times \frac{19}{7} \times \frac{7}{13} \times \frac{4}{3}$
$= \frac{19}{26}$.

121. If a number is placed before a bracket, *with no sign after it*, it is implied that the contents of the bracket are to be multiplied by the number. In like manner, if two brackets stand side by side, with no sign between them, it is implied that the contents of one bracket are to be multiplied by the contents of the other.

Ex. 9. Simplify $\dfrac{2\frac{1}{4} - \frac{2}{3} \text{ of } 1\frac{5}{6}}{\frac{1}{5} \text{ of } 3\frac{1}{3} + \frac{13}{36}}$

$$\frac{2\frac{1}{4} - \frac{2}{3} \text{ of } 1\frac{5}{6}}{\frac{1}{5} \text{ of } 3\frac{1}{3} + \frac{13}{36}} = \frac{2\frac{1}{4} - \frac{2}{3} \text{ of } \frac{11}{6}}{\frac{1}{5} \text{ of } \frac{10}{3} + \frac{13}{36}} = \frac{2\frac{1}{4} - \frac{11}{9}}{\frac{2}{3} + \frac{13}{36}} = \frac{36(2\frac{1}{4} - \frac{11}{9})}{36(\frac{2}{3} + \frac{13}{36})}$$
$$= \frac{81 - 44}{24 + 13} = \frac{37}{37} = 1.$$

NOTE 1.—In multiplying $2\frac{1}{4}$ by 36, multiply the fractional and integral parts separately, and add the results.

2. In multiplying $\frac{11}{9}$ by 36, divide the denominator, 9, into 36, and multiply the numerator, 11, by the quotient, 4.

Ex. 10. Simplify $8 + \dfrac{1}{2 + \dfrac{5}{7 + \dfrac{1}{2}}}$

Beginning at the lowest fraction, considering 5 as its numerator and $7 + \frac{1}{2}$ as its denominator, and multiplying each of these terms by 2, we have

$$8 + \frac{1}{2 + \dfrac{5}{7 + \dfrac{1}{2}}} = 8 + \frac{1}{2 + \dfrac{10}{14 + 1}} = 8 + \frac{15}{30 + 10} = 8\frac{3}{8}.$$

Exercise lxix.

Simplify the following expressions.

1. $\dfrac{3\frac{2}{3}}{4\frac{1}{7}} \times (3\frac{5}{8} \times 5\frac{4}{7}) - 17\frac{3}{4}$.
2. $(\frac{2}{3}$ of $\frac{5}{6}$ of $3\frac{3}{4} + 8\frac{2}{3}) \div (10\frac{1}{2} - 7\frac{5}{12})$.
3. $\dfrac{9\frac{5}{8}$ of $\frac{2}{11}}{6\frac{3}{10}$ of $\frac{5}{9}} + \dfrac{6\frac{2}{3}$ of $\frac{9}{10}}{4\frac{1}{8}$ of $2\frac{10}{11}}$.
4. $(19\frac{4}{5} - 3\frac{3}{4}) \times (3\frac{4}{5} - 2\frac{3}{7})$.
5. $19\frac{4}{5} - 3\frac{3}{4} \times 3\frac{4}{5} - 2\frac{3}{7}$.
6. $19\frac{4}{5} - 3\frac{3}{4} \times (3\frac{4}{5} - 2\frac{3}{7})$.
7. $\left(\dfrac{4\frac{1}{2} + 5\frac{3}{4}}{5\frac{7}{8} - 2\frac{1}{4}}\right) \times \left(2 - \dfrac{3}{5\frac{1}{8}}\right)$.
8. $\dfrac{\frac{4}{7}$ of $\frac{3}{8}}{5\frac{1}{14} - 4\frac{5}{56}} - \dfrac{5\frac{1}{4}$ of $\frac{13}{15}}{7\frac{1}{2}\frac{5}{2} - 6\frac{1}{2}}$.
9. $(\frac{1}{2} + \frac{1}{3}) \times (\frac{1}{4} + \frac{1}{6}) - (\frac{1}{2} - \frac{1}{3}) \times (\frac{1}{2} - \frac{1}{4})$.
10. $(\frac{1}{2} + \frac{1}{3}) \times (\frac{1}{4} - \frac{1}{5}) \div (\frac{1}{6} - \frac{1}{7})$.
11. $(\frac{2}{19} + \frac{1}{3}) \div (3 - \frac{1}{3}) \times (\frac{1}{3} + 2\frac{1}{6})$.
12. $(2\frac{1}{3} + 1\frac{1}{6} + 3\frac{1}{2}) \div \frac{1}{4}$ of $\frac{6}{5}$ of $1\frac{1}{2}$.
13. $\dfrac{1\frac{1}{4} - \frac{5}{12}}{1\frac{1}{4} + \frac{5}{12}} + \dfrac{6}{7}$ of $\dfrac{9 \times 5}{14 \times 3} - \dfrac{11\frac{1}{4}}{15}$.
14. $\dfrac{3\frac{1}{2} - 2\frac{1}{6}}{\frac{1}{4}$ of $(\frac{1}{5} + \frac{1}{7})} \div 15\frac{5}{9}$.
15. $2\frac{1}{2} + \dfrac{3\frac{1}{2} - \frac{1}{4}}{3\frac{1}{2} + \frac{1}{4}} - 2\frac{3}{5}$ of $1\frac{2}{13} - \frac{5}{7}$.
16. $2\frac{1}{2} + \dfrac{3\frac{1}{2} - \frac{1}{4}}{3\frac{1}{2} + \frac{1}{4}} - 2\frac{3}{5}$ of $(1\frac{2}{13} - \frac{5}{7})$.
17. $\frac{39}{72} \times \frac{24}{42}\frac{18}{5} \div \frac{32}{28}\frac{27}{9} \times \frac{75}{117}$.
18. $4\frac{1}{2}$ of $\frac{5}{27} \div 5\frac{3}{6}$ of $1\frac{3}{7}$.
19. $1\frac{1}{3}$ of $2\frac{1}{4} - 2\frac{1}{12} + 3\frac{3}{5} \times 3\frac{3}{8} + 12\frac{1}{7} \div 3\frac{3}{7}$.
20. $\dfrac{2\frac{1}{3}}{7} + \dfrac{4}{10\frac{1}{2}} - \dfrac{8}{15}$ of $\dfrac{3}{28}$.

EXAMINATION PAPERS.

I.

1. What is a fraction? Define a Simple, a Compound, a Proper Fraction.

2. What rule of fractions is anticipated in reducing a mixed number to an improper fraction?

3. If the numerator and denominator of a fraction be multiplied by the same number, the fraction thus obtained is equivalent to the former fraction. Prove the truth of this statement by taking the fractions $\frac{2}{3}$ and $\frac{10}{15}$ and showing that they are equivalent.

4. What name is given to a fractional expression of the form of $\frac{3}{5}$ of $\frac{7}{8}$?

State and illustrate the rule for multiplying one fraction by another, and show that the product of two proper fractions must always be numerically less than either of them?

5. What is meant by a vulgar fraction? When is a vulgar fraction greater than unity? What is it then called and why?

II.

1. A vulgar fraction may be considered as expressing the division of the numerator by the denominator. Explain this.

2. Explain the principle upon which vulgar fractions are reduced to their equivalents having a common denominator. When may the common denominator be less than the product of all the denominators; and how is it then determined? Ex. $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{5}{9}$, $\frac{11}{12}$.

3. By what fraction must $\frac{2}{3}$ be divided to give a quotient $\frac{11}{12}$? Can more than one such fraction be found?

4. State and prove the rule for the division of one vulgar fraction by another. Divide $\frac{3}{2}$ by $\frac{4}{5}$; show that a proper fraction will always be increased by dividing it by another proper fraction. By what fraction must $\frac{11}{12}$ be divided to give a quotient 3?

5. A man's wages are $\$3\frac{4}{5}$ a day, and his daily expenses

are $\$1\frac{7}{8}$; how many days must he labour to enable him to buy a suit of clothes worth $\$46\frac{1}{5}$?

III.

1. Define Numerator and Denominator, and show why they are properly applied to the terms of a fraction.

2. John had $\frac{3}{4}$ of a melon and gave away $\frac{5}{9}$ of what he had; what part of the melon had he left?

3. A miller wishes to put 39 bushels of wheat into bags, each bag to hold $2\frac{1}{6}$ bush.; how many bags would it require?

4. A man owned $\frac{3}{5}$ of a ship and sold $\frac{2}{3}$ of his share for $\$5175$; what was the whole ship worth?

5. If $7\frac{1}{2}$ pounds of coffee costs $187\frac{1}{2}$ cents; what will a bag containing $63\frac{2}{5}$ pounds cost?

IV.

1. Before subtracting fractions, why is it necessary to change them to others having a common denominator?

2. Arrange the fractions $\frac{3}{7}$, $\frac{2}{5}$, $\frac{7}{15}$, $\frac{13}{25}$, $\frac{2}{3}$ of $\frac{9}{16}$ in order of magnitude.

3. If $\$2\frac{2}{3}$ will pay a woman's wages for $2\frac{1}{2}$ days, how much will pay for $5\frac{1}{3}$ days' work?

4. James by mistake subtracted $\frac{5}{7}$ instead of $\frac{3}{7}$; was his answer too large, or too small, and how much?

5. A man owning $\frac{2}{3}$ of a factory, sold $\frac{2}{7}$ of what he owned for $\$15750$; what was the factory worth?

V.

1. State the principle involved when fractions are changed to others having a common denominator.

2. I bought $7\frac{5}{8}$ thousand feet of boards for $\$135.80$; at the same rate, what would $19\frac{3}{4}$ thousand feet cost?

3. I paid $\$7888.30$ for $83\frac{9}{16}$ acres of land; what would 7 acres cost at the same rate?

4. What is the least number that must be taken from 60 so that it may be exactly divisible by $7\frac{5}{8}$?

5. On $\frac{3}{4}$ of my field I planted corn; on $\frac{2}{3}$ of the remainder I sowed wheat; on $\frac{1}{2}$ of the remainder I planted potatoes; the rest, consisting of $\frac{1}{4}$ of an acre, was planted in beans; how large was my field?

CHAPTER V.

DECIMALS.

Section I.—Definitions.

Oral Exercises.

1. If an apple is divided into ten equal parts, what is one of the equal parts called? What are 7 of these called? 3 of them?

2. If a unit is divided into 10 equal parts, what are the parts called? What is the fractional unit?

3. If 1 tenth of an apple is divided into ten equal parts what part of the whole apple is 1 part? 3 parts? 9 parts?

4. How are hundredths got? How are they got from tenths?

5. What part of 1 tenth is 1 hundredth? How many 1 hundredths in 1 unit? In 1 tenth?

6. If 1 hundredth of an apple is divided into ten equal parts, what is the fractional unit called?

7. How many thousandths are equal to 1 hundredth? To 1 tenth? To 1 unit?

8. What is $\frac{1}{10}$ of $\frac{1}{10}$? $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$? $\frac{1}{10}$ of $\frac{1}{10\,0}$?

122. A Decimal Fraction is one which has for its denominator 10, 100, 1000, or *some power* of **ten.**

The **Power** of a number is the product obtained by multiplying the number *by itself* one, or more times.

Thus, 9 is the *second* power of 3, for $9 = 3 \times 3$.

27 " *third* " 3, for $27 = 3 \times 3 \times 3$.

81 " *fourth* " 3, for $81 = 3 \times 3 \times 3 \times 3$.

123. The *Denominator* of a decimal fraction is never expressed, but is always understood. For brevity decimal fractions are usually called *Decimals*. A decimal fraction

is expressed by writing the *Numerator* with a point (·) before it.

Thus, $\frac{1}{10}$ is written ·1.
$\frac{3}{10}$ " ·3.
$\frac{1}{100}$ " ·01.
$\frac{79}{100}$ " ·79.
$\frac{1}{1000}$ " ·001.
$\frac{139}{1000}$ " ·139.

124. The **Point** placed before decimals is called the *decimal point*. It separates the fractional part from whole numbers.

125. The *first* place to the right of the decimal point is that of **tenths**; the *second* place is that of **hundredths**; the *third*, that of **thousandths**; the *fourth*, that of **ten-thousandths**; the *fifth*, that of **hundred-thousandths**; etc.

Thus, $23{\cdot}045 = 2 \times 10 + 3 + \frac{0}{10} + \frac{4}{100} + \frac{5}{1000}$.

Hence it appears that decimals are simply an extension of the ordinary system of notation and numeration.

126. Naughts *affixed* to a decimal have no effect on its value; that is ·9, ·90, ·900 are all equal;

for, $\cdot 9 = \frac{9}{10}$.
$\cdot 90 = \frac{90}{100} = \frac{9}{10}$.
$\cdot 900 = \frac{900}{1000} = \frac{9}{10}$.

127. To convert a decimal to a vulgar fraction.

Since ·378 means 3 tenths, 7 hundredths, and 8 thousandths;

$$\therefore \cdot 378 = \frac{3}{10} + \frac{7}{100} + \frac{8}{1000}.$$
$$= \frac{300 + 70 + 8}{1000}$$
$$= \frac{378}{1000}.$$

Similarly ·00307 means 3 thousandths and 7 hundred thousandths;

$$\therefore \cdot 00307 = \frac{3}{1000} + \frac{7}{100000}$$
$$= \frac{300 + 7}{100000}$$
$$= \frac{307}{100000}.$$

Hence, *To express a decimal as a vulgar fraction write the given decimal as a whole number for the numerator*

of the vulgar fraction, and for the denominator write 1, followed by as many ciphers as there are decimal places in the given decimal.

Conversely, a fraction having 10, 100, 1000, etc., for denominator may be expressed as a decimal *by writing the numerator and counting off from the right as many figures as there are ciphers in the denominator.*

Thus $3\frac{175}{1000} = 3{\cdot}175$, and $\frac{75}{1000} = {\cdot}075$.

Exercise lxx.

Express the following decimals as common fractions:

1. ·7.	6. ·4123.	11. ·00427.
2. ·36.	7. ·0614.	12. ·00036.
3. ·08.	8. ·0078.	13. ·02007.
4. ·784.	9. ·7614.	14. ·712465.
5. ·709.	10. ·3005.	15. ·000006.

Express the following fractions as decimals:

16. $\frac{8}{10}$.	20. $\frac{136}{1000}$.	24. $126\frac{367}{1000}$.
17. $\frac{71}{100}$.	21. $2\frac{7}{100}$.	25. $\frac{18496}{100000}$.
18. $\frac{27}{100}$.	22. $4\frac{16}{100}$.	26. $3\frac{7}{100000}$.
19. $\frac{7}{100}$.	23. $16\frac{126}{1000}$.	27. $16\frac{163}{100000}$.

Exercise lxxi.

Write the following decimals in words:

1. ·9.	5. 4·31.	9. 21·3601.
2. ·27.	6. 7·216.	10. 17·0064.
3. ·368.	7. 3·314.	11. 18·00081.
4. ·064.	8. 5·8167.	12. 20·01458.

Express in figures the following:

13. Eight tenths; two, and seven hundredths; nine thousandths.

14. Eight hundred and seven, and ninety-four thousandths; three thousand and seventeen, and seven hundred and nine ten-thousandths; three, and one thousand and eight millionths.

15. Six, and four ten-thousandths; eighty, and six hundred and nine ten-millionths; one hundred and one, and one thousand and one hundred-thousandths.

Section II.—Addition.

128. To add Decimals.

Ex. 1. What is the sum of 3·7, 14·035, 81·64 and ·7165?

3·7000		3·7
14·0350		14·035
81·6400	or	81·64
·7165		·7165
100·0915		100·0915

Since we can add numbers of the same name only, we write the addends so that units will be under units, tenths under tenths, etc. This is always the case when the points range in the same straight line. Then, beginning at the lowest order, we add as if the figures were integers and place the decimal point in the sum before the tenths.

Exercise lxxii.

(1)	(2)	(3)	(4)
42·3	12·326	4031·06	·608242
13·06	204·00	108·304	·0315044
8·049	8·3024	9·001345	·8034
1·6	52·007	76·739	·086
·037	324·1	250·0007	·9106

Find the sum of

5. 4·5 + 70·63 + 1·079 + 25.
6. ·126 + 3·05 + ·07 + ·528 + 7·093.
7. 111·306 + ·0317 + 2·793 + ·007.
8. 470·05 + 72·701 + 3·0315 + 413·2658.
9. 12·3987 + 4·1462 + ·02063 + 13 + 10·962.
10. 210·7 + 14563·21 + ·0173 + 382·74156.
11. 9·127 + 17·72 + ·0041 + 2·31 + 170·96.
12. ·101285 + 17·061 + 3·2001 + 5·38706.
13. 2·325 + ·0012 + 5·086 + 219·6832 + ·407.

14. A merchant has 4 pieces of calico measuring respectively 25·5 yards, 29·125 yards, 34·25 yards, and 33·75 yards; how many yards are there in the 4 pieces.

15. Four fields contain as follows: 15·375 acres, 12·6125 acres, 14·003 acres, 16·5 acres; how many acres do the four fields contain?

Section III.—Subtraction.

129. To subtract Decimals.

Ex. 1. From 17·013 take 1·90764.

17·01300		17·013
1·90764	or	1·90764
15·10536		15·10536

We write the subtrahend under the minuend, placing tenths under tenths, hundredths under hundredths, etc. Then, as there are more figures in the subtrahend than in the minuend, we may annex as many ciphers as will render the number of decimal places in each the same. This will not affect the value of the minuend (Art. 126). We then subtract as in whole numbers and place the decimal point in the remainder immediately to the left of the tenths.

Exercise lxxiii.

	(1)	(2)	(3)	(4)
From	18·5	2·8706	·50376	·36
Take	2·3476	·49	·065	·12704

From

5. 1·869	take	·0374.	9. 204·1	take	36·002.
6. ·0061	"	·00089.	10. 1000	"	999·99.
7. 6·723	"	2·7981.	11. 2	"	1·3678.
8. 9·305	"	7·9.	12. 17·36	"	9·0184.

Find the value of

13. $(7·2 - 2·75) - (1·9 - ·0027)$.

14. $36 + 7·07 - 24·896 - (3·164 - ·799)$.

15. $(273·29 - 41·802) - (7·162 + 51·386 - ·09863)$.

16. The length of a second's pendulum is 39·1392 inches, and that of a French metre 39·371 inches. Find the difference in length between them.

17. A sovereign weighs 123·274 grains, and a shilling 87·272 grains; find their difference in weight.

18. Take eleven thousandths from eleven hundredths.

19. Add together the sum and difference of seventy-three thousandths and one hundred and fifteen millionths.

20. From a piece of muslin containing 27·5 yards, a merchant sold 13·75 yards; how much was left?

Section IV.—Multiplication.

130. To multiply Decimals.

Ex. 1. Multiply ·7 by ·9.

Since $\cdot 7 = \frac{7}{10}$ and $\cdot 9 = \frac{9}{10}$;

$\therefore \cdot 9 \times \cdot 7 = \frac{9}{10} \times \frac{7}{10} = \frac{63}{100} = \cdot 63.$

Ex. 2. Multiply ·731 by ·06.

Since $\cdot 731 = \frac{731}{1000}$ and $\cdot 06 = \frac{6}{100}$;

$\therefore \cdot 06 \times \cdot 731 = \frac{6}{100} \times \frac{731}{1000} = \frac{4386}{100000} = \cdot 04386.$

Ex. 3. Multiply 3·76 by 2·4.

Since $3 \cdot 76 = \frac{376}{100}$ and $2 \cdot 4 = \frac{24}{10}$;

$\therefore 2 \cdot 4 \times 3 \cdot 76 = \frac{24}{10} \times \frac{376}{100} = \frac{9024}{1000} = 9 \cdot 024.$

Hence, *To multiply decimals, multiply as in the case of integers and mark off from the right of the product as many decimal places as there are decimals in the factors.*

Exercise lxxiv.

	(1)	(2)	(3)	(4)
Multiply	4·64	53·062	·1346	675·1
By	3·35	4·53	·203	·008

Multiply

5. 713 by 3·47.
6. 3·96 by ·068.
7. 9·07 by 1·06.
8. ·008 by ·009.
9. 13·14 by ·0236.
10. 714·6 by 1·124.
11. 9·006 by ·0045.
12. 1·001 by 1·009.

13. A square link contains 62·726 square inches; what is the area in inches of 5327 square links?

14. A pint of water weighs 1·25 pounds avoirdupois; what is the weight of 7·8 pints?

15. Gold is 19·26 times as heavy as water; what weight of gold is of the same bulk as 17·342 pounds of water?

16. The circumference of a circle measures 3·14159 times its diameter; what will be the length of the circumference of a circle whose diameter measures 37·258 miles?

17. Find the product of the sum and difference of ·27 and 27.

18. What is the weight of five cubic feet of water if a cubic foot weighs 62·455 pounds avoirdupois?

131. To multiply by 1 followed by ciphers.

Ex. 4. Multiply 71·134 by 10; by 100; by 100000.

71·134	71·134	71·134
10	100	100000
711·340	7113·400	7113400·000

From these examples, it will be seen that the decimal point has been moved to the *right* in the product as many places as there are ciphers in the multiplier.

Hence, *To multiply by 1 followed by ciphers, move the decimal point as many places to the right in the multiplicand as there are ciphers in the multiplier, and the result will be the product.*

Section V.—Division.

132. To divide one decimal by another.

Ex. 1. Divide 9 by ·3.

$$\cdot\frac{9}{3} = \frac{10 \times 9}{10 \times \frac{3}{3}} = \frac{90}{3} = 30.$$

In this example we multiply both divisor and dividend by 10. This makes the divisor a whole number. We then proceed as in ordinary division.

Ex. 2. Divide 97·92 by 9.

9)97·92 As the divisor is already a whole number
10·88 we proceed to divide as usual.

Ex. 3. Divide 3·24 by ·00081.

Multiplying both divisor and dividend by 100000 we get $324000 \div 81$, which can easily be worked by ordinary division.

Ex. 4. Divide ·736644 by 234·6.

We multiply the divisor and dividend by 10; the divisor is now a whole number. The operation will then stand as follows:

2346)7·36644(·00314
7·038
3284
2346
9384
9384

We first bring down 3 tenths and put the decimal point in the quotient. The divisor is not contained in 73 tenths; we therefore put a 0 in the quotient and bring down 6 hundredths. Since the divisor is not contained in 736 hundredths, we put another 0 in the quotient and bring down 6 thousandths. The divisor is now contained in 7366 thousandths. The rest of the work proceeds as in ordinary division.

Hence, *If the divisor is a decimal, we multiply both divisor and dividend by such a power of 10 as will make the divisor a whole number, and then we divide as in simple division, placing the decimal point in the quotient as soon as the tenths figure of the dividend is brought down.*

Exercise lxxv.

Divide

1. 16·578 by 5·4.
2. 48·591 by ·96.
3. 2·56 by ·0032.
4. 4·126 by 640.
5. 3·1 by ·0025.
6. ·0012 by 1·6.
7. ·0774 by 480.
8. 21·3 by 37·5.
9. 202 by ·01.
10. 406·8 by ·018.
11. 1·066 by 13.
12. 15·77 by ·19.

133. To divide by 1 followed by ciphers.

Ex. 4. Divide 136·15 by 10; by 100; by 10000.

10 | 136·15	100 | 136·15	10000 | 136·15
13·615	1·3615	·013615

From these examples it will be seen that the decimal point has been moved to the *left* in the quotient as many places as there are ciphers in the divisor.

Hence, *To divide by 1 followed by ciphers, move the decimal point as many places to the left in the dividend as there are ciphers in the divisor, and the result will be the quotient.*

Section VI.—Reduction of Decimals.

134. To reduce a Vulgar Fraction to a Decimal.

Ex. 1. Reduce $\frac{3}{40}$ to a decimal.

40)300(·075
280
200
200

$\frac{3}{40}$ equals $\frac{1}{40}$ of 3 (Art. 94). 3 equals 30 tenths, and $\frac{1}{40}$ of 30 tenths is 0 tenths. 30 tenths equals 300 hundredths, and $\frac{1}{40}$ of 300 hundredths is 7 hundredths, and 20 hundredths remaining. 20 hundredths equals 200 thousandths, and $\frac{1}{40}$ of 200 thousandths is 5 thousandths; hence $\frac{3}{40} = ·075$.

Hence, *To reduce a vulgar fraction to a decimal annex ciphers to the numerator and divide by the denominator of the fraction, and place the decimal point in the quotient as soon as the first cipher annexed is brought down.*

Exercise lxxvi.

Reduce the following to decimals:

1. $\frac{3}{16}$.
2. $\frac{3}{4}$.
3. $\frac{5}{8}$.
4. $\frac{9}{40}$.
5. $\frac{5}{32}$.
6. $\frac{1}{40}$.
7. $\frac{3}{80}$.
8. $\frac{7}{8}$.
9. $\frac{11}{160}$.
10. $\frac{5}{64}$.
11. $\frac{1}{125}$.
12. $6\frac{3}{5}$.
13. $24\frac{1}{125}$.
14. $3\frac{21}{40}$.
15. $46\frac{5}{16}$.

Section VII.—Circulating Decimals.

135. To reduce a circulating decimal to a vulgar fraction.

In reducing vulgar fractions to decimals we find that sometimes the division will not terminate, but the same figure or figures will be repeated over again, continually.

Ex. 1. Reduce $\frac{1}{3}$ to a decimal.

$\frac{1}{3} = \cdot 3333$, etc.

Ex. 2. Reduce $\frac{5}{11}$ to a decimal.

$\frac{5}{11} = \cdot 4545$, etc.

136. Decimals of this kind are called **Repeating** or **Circulating Decimals.** The part repeated is called the **Period** or **Repetend.**

137. It is usual to express the repetend by writing it down and placing a dot over the first and last figures of the part repeated. When there is only one figure repeated the dot is placed over it.

Thus, $\cdot 3333$, etc., is indicated $\cdot\dot{3}$.

$\cdot 4545$, etc., " $\cdot\dot{4}\dot{5}$.

$\cdot 2333$, etc., " $\cdot 2\dot{3}$.

Ex. 3. Reduce $\frac{3}{22}$ to a decimal.

$\frac{3}{22} = \cdot 1363636$, etc., $= \cdot 1\dot{3}\dot{6}$.

138. A pure circulating decimal is one in which the figures that repeat begin immediately after the decimal point.

139. A mixed circulating decimal is one in which the figures that repeat do not begin immediately after the decimal point.

140. Since

$\frac{1}{9} = \cdot 11111\ldots$ Also $\frac{1}{99} = \frac{1}{9} \div 11 = \cdot 010101\ldots$

$\frac{2}{9} = \cdot 22222\ldots$ $\frac{5}{99} = \cdot 050505\ldots$

$\frac{5}{9} = \cdot 55555\ldots$ $\frac{17}{99} = \cdot 171717\ldots$

Similarly, $\frac{1}{999} = \frac{1}{9} \div 111 = \cdot 001001\ldots$

$\frac{125}{999} = \cdot 125125\ldots$

From the preceding examples it is evident that a *Pure Circulating Decimal may be expressed as a fraction by writing the figures that repeat as numerator, and as many nines as there are figures in the repetend for denominator of the fraction.*

Thus, $\cdot\dot{0}\dot{5} = \frac{5}{99}$. $\quad \cdot\dot{3}\dot{7}\dot{8} = \frac{378}{999}$.

$\cdot 5\dot{4} = \frac{54}{99}$. $\quad 5\cdot 4\dot{3} = 5\frac{43}{99}$.

$3\cdot 4 = 3\frac{4}{9}$. $\quad \cdot 0\dot{3}7\dot{8} = \frac{378}{9999}$.

141. Mixed Circulating Decimals may be reduced to vulgar fractions in the following manner:

Ex. 4. $\cdot 03\dot{4} = \cdot 03\frac{4}{9} = \frac{3\frac{4}{9}}{100} = \frac{31}{900} = \frac{34-3}{900}$.

Ex. 5. $\cdot 05\dot{4}\dot{3} = \cdot 05\frac{43}{99} = \frac{5\frac{43}{99}}{100} = \frac{538}{9900} = \frac{543-5}{9900}$.

Ex. 6. $\cdot 013\dot{6} = \cdot 013\frac{6}{9} = \frac{13\frac{6}{9}}{1000} = \frac{123}{9000} = \frac{136-13}{9000}$.

From these examples it is evident that a *Mixed Circulating Decimal may be expressed as a fraction by subtracting the part of the decimal which does not repeat from the whole decimal and placing the remainder as numerator, and as many nines as there are figures in the repetend, followed by as many ciphers as there are figures in the part which does not repeat, as denominator of the fraction.*

Exercise lxxvii.

Reduce to vulgar fractions:

1. $\cdot\dot{3}$.	5. $\cdot 0\dot{2}\dot{4}$.	9. $4\cdot 053\dot{1}$.
2. $\cdot 5\dot{4}$.	6. $\cdot 3\dot{1}\dot{4}$.	10. $11\cdot 28\dot{7}$.
3. $\cdot\dot{7}2\dot{9}$.	7. $\cdot 00\dot{6}7\dot{5}$.	11. $3\cdot\dot{4}1\dot{8}$.
4. $\cdot\dot{3}2\dot{9}$.	8. $\cdot 044\dot{3}$.	12. $2\cdot 3\dot{4}5$.

142. The **Addition** or **Subtraction** of **Circulating Decimals** is generally performed by repeating the period as many times as seems sufficient to insure the required degree of accuracy, and then adding or subtracting.

143. Multiplication or **Division** of **Circulating Decimals** may also be performed by carrying out the repetend, but these operations are more usually performed by reducing the decimals to vulgar fractions, then multiplying or dividing these fractions, and reducing the results once more to decimals.

Ex. 7. Multiply $\cdot 2\dot{3}$ by $\cdot \dot{3}\dot{6}$.

$$\cdot \dot{3}\dot{6} \times \cdot 2\dot{3} = \tfrac{3\,6}{9\,9} \times \tfrac{2\,1}{9\,0} = \tfrac{1\,4}{1\,6\,5} = \cdot 0\dot{8}4.$$

Ex. 8. Divide $\cdot 1\dot{6}$ by $\cdot 00\dot{2}\dot{7}$.

$$\cdot 1\dot{6} \div \cdot 00\dot{2}\dot{7} = \tfrac{1\,5}{9\,0} \div \tfrac{2\,7}{9\,9\,0\,0} = \tfrac{5\,5\,0}{9} = 61\cdot\dot{1}.$$

Exercise lxxviii.

Find the value of

1. $\cdot 3\dot{1}00\dot{7} + 21\cdot 00\dot{3} + 41\cdot 60\dot{7}3\dot{4}\dot{2}$.
2. $\cdot \dot{3} - \cdot 0\dot{9}$ and $\cdot 0\dot{4} - \cdot 007692\dot{3}\dot{8}$.
3. $37\cdot 2\dot{3} \times \cdot 2\dot{6}$ and $7\cdot 7\dot{2} \times \cdot 2\dot{9}\dot{7}$.
4. $\cdot \dot{3} \div \cdot 0\dot{9}$ and $\cdot 042 \div \cdot 03\dot{6}$.

EXAMINATION PAPERS.

I.

1. What are decimal fractions? How does the use of them facilitate calculation?

2. Represent as vulgar fractions 1·25, ·0001. How does it affect the value of a decimal to place ciphers (1) after the decimal places, (2) between the decimal places and the decimal point. Decimals may be multiplied and divided by 10, 100, 1000, etc., merely by shifting the decimal point; show this. Divide ·000121 by 11.

3. What are the advantages of decimal fractions? Express as a decimal, 17359 divided by one million. Divide ·00125 by 2·5. If the number of decimal places in the divisor exceeds the number in the dividend, how do you proceed? Explain this by making 2·5 the dividend and ·00125 the divisor.

4. Multiply 2·564 by ·047, and divide ·00169 by ·013. Verify the result by putting the decimals in the form of vulgar fractions.

5. What are recurring decimals? Find the recurring decimal equivalent to $\frac{5}{7}$, and find the vulgar fraction equivalent to the recurring decimal ·81246246....

II.

1. Explain the notation of decimal fractions, and show how the value of a decimal is affected by moving the decimal point two places to the right or left. Write $\frac{375}{1000}$ as a decimal, and express the one-millionth part of the same fraction as a decimal. Multiply 85·345 by 4·175. Divide 25·6 by ·00016.

2. Divide ·365 by 20. If in obtaining the quotient you cut off the cipher from the divisor and actually divide by 2, what corresponding change should be made in the dividend?

3. Prove that $\cdot\dot{3}3\dot{3} \times \cdot 2121\dot{2}\dot{1} = \cdot 070\dot{7}0\dot{7}$.

4. Prove the rule for fixing the position of the decimal point, when one decimal fraction is multiplied by another.

Express as vulgar fractions in their lowest terms:

(1) ·0625 × ·0032; (2) ·016 ÷ ·64; ·45 − ·45.

5. Simplify $\frac{1\cdot18}{\cdot152} \times \frac{3\cdot04}{2\cdot95}$, and divide the result by ·00125.

III.

1. Prove the rule for dividing one decimal fraction by another, and find the value of $\frac{\cdot05 \times \cdot05 \times \cdot05 + 1}{1\cdot05}$.

2. State and explain the rule for reducing a vulgar fraction to a decimal fraction.

Find the value of $\frac{1}{4} \div \cdot01001$ and of $10\cdot01 \div \frac{1}{50}$.

3. Reduce ·064 and 15·625 to vulgar fractions; multiply them together in that form, and then reduce the result to decimals. Prove by multiplying the decimals as they stand.

4. Which is the greater, $1\frac{2}{3} \times 2\frac{5}{9}$, or $\cdot 018 \times 216$?

5. Suppose unity represents $\cdot 0012$, what number represents $\cdot 0001$.

IV.

1. Whether is $1\cdot 121472653$ more accurately represented by $1\cdot 1214726$ or $1\cdot 1214727$, and why?

2. Express in decimal notation the value of $8\cdot 0625 - 6\frac{1}{2\,5} - \cdot 00375 + 1\cdot 09236 - \frac{2\,5\,6\,7\,9}{1\,0\,0\,0\,0}$.

3. *A* bought a house with $\cdot 25$ of his money; he spent $\cdot 575$ of it in buying a farm and had \$2100 left; find the cost of the house and farm respectively.

4. What is the smallest number that can be exactly divided by the nine significant figures?

Simplify $\dfrac{\frac{5}{8}\text{ of }\frac{4}{1\,0} - \frac{7}{9}\text{ of }\frac{4}{2\,1}}{\frac{5}{6}\text{ of }\frac{2}{1\,5} - \frac{3}{8}\text{ of }\frac{4}{1\,8}}$.

5. What number is that, from which if there be taken $\frac{2}{7}$ of $\cdot 375$, and to the remainder $\cdot 53$ of $\cdot 3125$ be added, the sum is 10?

V.

1. Find the value of $\frac{3}{1\,1}$ of $(\frac{7}{5} + 1\frac{2}{3})$ and prove it equal to $\frac{1}{3}$ of $20\frac{3}{4} \div 10\frac{3}{8}$.

2. Prove the rule for finding the value of a circulating decimal; and reduce $1 \div 99999$ and $1 \div 10001$ to circulating decimals.

3. Prove that $46\cdot 2 \div 92\cdot 4 = \cdot 75 \times \cdot\dot{6}$.

4. Prove that $\cdot 02 \times \cdot 02 \times \cdot 005 \times \cdot 005 = \cdot 0001 \times \cdot 0001$.

5. Divide $\frac{1}{4} + \frac{1}{2} + \frac{1}{1\,5} + \frac{1}{1\,8} + \frac{1}{2\,0}$ by $\frac{1}{6} + \frac{1}{1\,0} + \frac{1}{1\,8} + \frac{1}{3\,0}$, and reduce the result to a decimal.

CHAPTER VI.

COMMERCIAL ARITHMETIC.

Section I.—Tables and Reduction.

144. ENGLISH OR STERLING MONEY.

4 farthings (far.) - - - = 1 penny, or 1d.
12 pence - - - - - - = 1 shilling, " 1s.
20 shillings - - - - - = 1 pound, " £1.

NOTE 1.—Farthings are usually written as a fraction of 1d. Thus 1 far. is written $\frac{1}{4}$d.; 2 far., $\frac{1}{2}$d.; 3 far., $\frac{3}{4}$d.

NOTE 2.—£1 sterling = \$4.86$\frac{2}{3}$, and 1s. = 24$\frac{1}{3}$ cents.

Oral Exercises.

Repeat the table of English money.
How many far. in 2d.? in 3d.? in 6d.? in 8d.?
How many pence in 12 far.? in 16 far.? in 20 far.?
How many pence in 2s.? in 3s.; in 5s.; in 6s.?
How many far. in 1s.? in 2s.? in 3s.? in 5s.?
How many shillings in £1 12s.? in £2 15s.?

145. There are two kinds of **Reduction, Reduction Descending** and **Reduction Ascending.**

146. Reduction Descending is the process of changing a number from a *higher* to a *lower* denomination.

147. Reduction Ascending is the process of changing a number from a *lower* to a *higher* denomination.

Ex. 1. Reduce £6 5s. 3$\frac{1}{4}$d. to farthings.

£6 5s. 3$\frac{1}{4}$d.
20
125s.
12
1503d.
4
6013 far.

In 1 pound there are 20 shillings, and in £6 there are 6 times 20s., or 120s.; 120s. plus 5s. are 125s.; in 1 shilling there are 12 pence, and in 125s. there are 125 times 12d., or 1500d.; 1500d. plus 3d. are 1503d.; in 1d. there are 4 farthings, and in 1503d. there are 1503 times 4 far., or 6012 far.; 6012 far. plus 1 far. are **6013 far.**

Ex. 2. How many £. s. d. in 3679 farthings?

far.
4 | 3679
12 | 919 3 far.
20 | 76 7d.
£3 16s.
Ans. £3 16s. $7\frac{3}{4}$d.

There are 4 far. in 1d.; hence, in 3679 far. there are as many pence as the number of times 4 is contained in 3679; $3679 \div 4 = 919$ and 3 over. This 3 is 3 far. There are 12d. in 1s.; hence, in 919d. there are as many shillings as the number of times 12 is contained in 919; $919 \div 12 = 76$ and 7 over. This 7 is 7d. There are 20s. in £1; hence, in 76s. there are as many pounds as the number of times 20 is contained in 76; $76 \div 20 = 3$ and 16 over. This is 16 shillings.

Exercise lxxix.

Reduce

1. 7s. 8d. to pence.
2. £1 3s. to farthings.
3. 7145d. to £., etc.
4. 6185s. to £., etc.
5. £10 0s. 6d. to pence.
6. £2 6s. 8d. to pence.
7. 3910 far. to £., etc.
8. 7163d. to £., etc.
9. £191 9s. $11\frac{3}{4}$d. to far.
10. £3 6s. $10\frac{1}{4}$d. to far.
11. 78916d. to £., etc.
12. £100 7d. to far.

148. UNITED STATES MONEY.

10 mills (*m.*) - - - - - - - - - = 1 cent, or 1 c.
10 cents - - - - - - - - - - = 1 dime, " 1 d.
10 dimes - - - - - - - - - - = 1 dollar, " 1 $.
10 dollars - - - - - - - - - = 1 eagle, " 1 e.

149. AVOIRDUPOIS WEIGHT.

16 ounces - - - - - = 1 pound - - - - or 1 lb.
100 pounds - - - - - 1 hundred-weight - " 1 cwt.
20 hundred-weight - - = 1 ton - - - - - " 1 t.

NOTE 1. — Avoirdupois Weight is used for weighing everything except jewels, precious metals, and medicines when dispensed.

NOTE 2.—112 pounds equal 1 cwt. in Great Britain.

Oral Exercises.

Repeat the table of Avoirdupois Weight.
How many ounces in 2 lb.? in 3 lb. 4 oz.? in 4 lb.?
How many pounds in 6 tons? 8 tons? 3 t. 500 lb.
What part of a pound is 2 oz.? 8 oz.? 12 oz.?
How many tons in 58 cwt.? in 112 cwt.? in 200 cwt.

Ex. 3. Reduce to ounces 3 t. 6 cwt. 51 lb. 7 oz.

20
66 cwt.
100
6651 lb.
16
106423 oz.

Ex. 4. Reduce 147658 lb. to tons, etc.

100 | 147658 lb.
20 | 1476 cwt. 58 lb.
73 t. 16 cwt. 58 lb.

Exercise lxxx.

Reduce

1. 2 t. 81 lb. to ounces.
2. 5 lb. 6 oz. to ounces.
3. 21645 oz. to cwt., etc.
4. 76385 oz. to tons, etc.
5. 3 cwt. 8 lb. 5 oz. to ounces.
6. 51649 lb. to tons, etc.

150. TROY WEIGHT.

24 grains (gr.) - - = 1 pennyweight, - - or 1 dwt.
20 pennyweights - - = 1 ounce, - - - - - " 1 oz.
12 ounces - - - - = 1 pound, - - - - - " 1 lb.

NOTE 1.—This is chiefly used for weighing gold, silver and jewels.

NOTE 2.—7000 Troy grains = 1 lb. Avoirdupois.
5760 " " = 1 lb. Troy.
437½ " " = 1 oz. Avoirdupois.
480 " " = 1 oz. Troy.

Oral Exercises.

How many oz. in 2 lb.? in 3 lb.? in 5 lb.?
How many lb. in 36 oz.? in 48 oz.? in 60 oz.? in 44 oz.? in 78 oz.?
How many dwt. in 2 oz.? in 3 oz.? in 4 oz.? in 48 gr.?

151. APOTHECARIES' WEIGHT.

20 grains (gr.)	= 1 scruple,	or 1 sc. or 1 ℈.
3 scruples	= 1 dram,	" 1 dr. " 1 ʒ.
8 drams	= 1 ounce,	" 1 oz. " 1 ℥.
12 ounces	= 1 pound,	" 1 lb. " 1 ℔.

NOTE 1.—The ounce and pound of Apothecaries' Weight are the same as in Troy Weight.

NOTE 2.—Druggists buy their medicines by Avoirdupois Weight and retail them by Apothecaries' Weight.

How many

1. Grains in 7 ℈? 11 ℈?
2. Scruples in 9 ʒ? 16 ʒ?
3. Drams in 24 ℈? 96 ℈?
4. Drams in 5 ℥? 7 ℥?
5. Ounces in 88 ʒ? 96 ʒ?
6. Pounds in 108 ℥? 168 ℥?

Exercise lxxxi.

Reduce

1. 1 lb. 4 oz. to ounces.
2. 7163 sc. to lb., etc.
3. 7685 dwt. to lb., etc.
4. 11 oz. 3 drs. to grains.
5. 3 oz. 6 dwt. to grains.
6. 73564 grains to lb. (Troy) etc.

152. LONG MEASURE.

12 inches (in.)	= 1 foot,	or 1 ft.
3 feet	= 1 yard,	" 1 yd.
5½ yards	= 1 rod,	" 1 rd.
320 rods	= 1 mile,	" 1 mi.

NOTE 1.—The mile used in the foregoing table is called a *Statute Mile*. The geographical or nautical mile, also called a *knot*, is equal to 1·15 statute miles. The knot is used in estimating the speed of vessels.

NOTE 2.—Gunter's Chain is used in measuring land. It is 22 yards in length and is divided into 100 links, each link being 7·92 inches long.

NOTE 3.— 6 feet = 1 fathom.
120 fathoms = 1 cable length.
880 fathoms = 1 mile.

NOTE 4.—The Hand (the breadth of the hand and thumb) used in measuring the height of horses at the shoulder is 4 inches.

Oral Exercises.

Repeat the table of Lineal Measure.

How many feet in 4 yd.? in 6 yd. 1 ft.?

How many inches is this page in width? in length?

How many yards, around this room? How many feet? How many inches?

Ex. 5. How many feet in 12 rd. 3 yd. 2 ft.?

rd.	yd.	ft.
12	3	2
$5\frac{1}{2}$		
63		
6		
69 yd.		
3		
209 ft. *Ans.*		

Ex. 6. How many rods in 209 ft.?

feet

3(209

$5\frac{1}{2}$)69 yd....2 ft.

2 2

11)138

12..6 half yd. = 3 yd.

Ans. 12 rd. 3 yd. 2 ft.

Exercise lxxxii.

Reduce

1. 1 mi. 9 rd. to inches.
2. 76452 in. to mi., etc.
3. 7568 ft. to mi., etc.
4. 2 rd. 1 yd. to feet.
5. 7 chains to feet.
6. 16752 in. to fathoms.

153. SURFACE OR SQUARE MEASURE.

144 square inches = 1 square foot, or 1 sq. ft.
9 square feet = 1 square yard, " 1 sq. yd.
$30\frac{1}{4}$ square yards = 1 square rod, " 1 sq. rd.
160 square rods = 1 acre, - - " 1 a.
640 acres - - - = 1 square mile " 1 sq. mi.

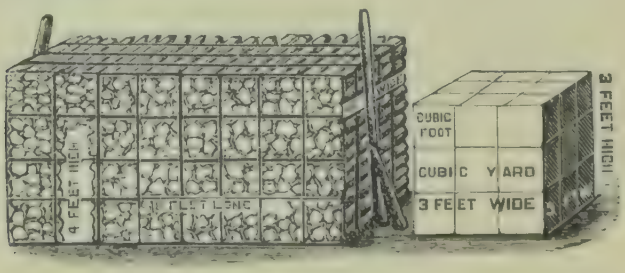
NOTE 1.—10,000 square links = 1 square chain.
10 square chains = 1 acre.

NOTE 2.—160 a. = a quarter-section land.
320 a. = a half-section land.
640 a. = a whole-section land.

154. CUBIC OR SOLID MEASURE.

1728 cubic inches (cu. in.) = 1 cubic foot, or 1 cu. ft.
27 cubic feet - - - = 1 cubic yard, " 1 cu. yd.
128 cubic feet - - - = 1 cord, - - " 1 cd.

NOTE 1.—A cord is a pile of wood 4 ft. wide, 4 ft. high and 8 ft. long.



Oral Exercises.

Repeat the table of square measure.
Write a table of cubic measure.
How many inches in 2 sq. ft.?
How many acres in 1600 sq. rods? in 320 sq. rods?
How many feet in 3 cu. yd.? in 4 cu. yd. 20 cu. ft.?
What is the difference between 3 sq. in. and 3 in. square?
3 in. square is a square each side of which is 3 in. long, and hence = 9 sq. in.

Exercise lxxxiii.

Reduce

1. 1997 sq. rd. to acres, etc.
2. 3 sq. rd. 2 ft. to inches.
3. 8469 cu. in. to feet.
4. 7689 cu. ft. to cords.
5. 12 a. 6 rd. to inches.
6. 78 cu. ft. 640 in. to inches.

155. DRY MEASURE.

2 pints (pt.) - - - - - - - - - = 1 quart, or 1 qt.
4 quarts - - - - - - - - - - = 1 gallon, " 1 gal.
2 gallons - - - - - - - - - - = 1 peck, " 1 pk.
4 pecks - - - - - - - - - - = 1 bushel, " 1 bu.

NOTE 1.—Dry Measure is used in measuring grain, fruit, etc.

NOTE 2.—By the "Weights and Measures" Act of 1873, the "Imperial Bushel," containing eight "Imperial Gallons," of 277·274 cubic inches in each gallon, is the standard bushel in Canada.

NOTE 3.—By the same Act the following articles are to be estimated by the *Cental* of 100 lbs.: Barley, beans, charcoal, corn, oats, peas, potatoes, rye, salt, seeds and wheat.

NOTE 4.—8 bushels = 1 quarter in Great Britain.

NOTE 5.—The following table shows the weight of a bushel of the article named, as determined by the same Act:

Beans - - 60 lb.	Wheat - 60 lb.	Barley - - 48 lb.
Clover seed 60 lb.	Corn - 56 lb.	Buckwheat 48 lb.
Peas - - 60 lb.	Rye - - 56 lb.	Timothy seed 48 lb.
Potatoes - 60 lb.	Flax seed 56 lb.	Oats - - - 34 lb.

156. LIQUID MEASURE.

2 pints - - - - - - - - - - - = 1 quart or 1 qt.
4 quarts - - - - - - - - - - - = 1 gallon, " 1 gal.

NOTE 1.—A barrel of beer - - contains 36 gals.
A hogshead of wine - " 63 gals.

NOTE 2.—The *wine gallon* contains 231 cubic inches.

NOTE 3.—6 wine gals. = 5 standard gals.

NOTE 4.—A gallon of pure water 62° F. weighs 10 lb.

157. MEASURE OF TIME.

60 seconds (sec.) - - - - - - = 1 minute, or 1 min.
60 minutes - - - - - - - - - = 1 hour, " 1 h.
24 hours - - - - - - - - - - = 1 day, " 1 da.
7 days - - - - - - - - - - - = 1 week, " 1 wk.
12 calendar months or 365 days - = 1 year, " 1 yr.
366 days - - - - - - - - - - = 1 leap year.

NOTE 1.—The number of days in each month may be remembered by means of the following lines:

Thirty days has September,
April, June, and November;
February has twenty-eight alone—
All the rest have thirty-one:
But leap year coming once in four
February then has one day more.

NOTE 2.—The leap years are those that can be divided by 4 without a remainder: as, 1884, 1888, 1892, etc. But of the even hundreds, only those that can be divided by 400 are leap years. The year 1900 will not be a leap year, but 2000 will be.

158. CIRCULAR OR ANGULAR MEASURE.

60 seconds (″) - - - - - - - - - - = 1 minute (′).
60 minutes - - - - - - - - - - - = 1 degree (°).
360 degrees - - - - - - - - - - - = 1 circumference.

NOTE 1.—*Circular or Angular Measure* is used to measure arcs of circles, angles, and in determining latitude, longitude, direction, the position of vessels at sea, etc.

NOTE 2.—A degree of the circumference of the earth at the equator contains 60 geographical miles, or 69·16 statute miles.

159. MISCELLANEOUS TABLE.

12 units = 1 dozen.	24 sheets - = 1 quire.
12 dozen = 1 gross.	20 quires - = 1 ream.
12 gross = 1 great gross.	196 lb. flour - = 1 barrel.
20 units = 1 score.	200 lb. pork - = 1 barrel.

Oral Exercises.

Repeat Time Measure.
How many days in 3 weeks? in 5 weeks and 3 days?
How many dozen in 84? in 132? in 150?
Was 1600 a leap year? 1876? 1854?
How many hours in 360 min.? in 788 min.? 600 min.?

Exercise lxxxiv.

Reduce

1. 7 da. 16 hr. to seconds.
2. 7684 pints to bushels, etc.
3. 84 gal. 3 quarts to pints.
4. 36 bu. 3 qt. 1 pt. to pints.

Reduce

5. 2685 pints to gallons.
6. 17 qr. 3 bu. to pecks.
7. 3685 lb. of wheat to bu.
8. 785693 sec. to weeks, etc.
9. 3586 lb. Timothy seed to bu., etc.
10. 78 da. 9 min. to seconds.
11. 1576 cu. ft. to cords.

12. If the 1st July be on Monday, on what day of the week will Christmas day fall?

Days from 1st July till 25th Dec. = 177 days.
Weeks " " " = 25 weeks, 2 days.
Therefore 2 days from Monday is Wednesday.
Christmas day will, therefore, fall on Wednesday.

13. The 2nd January, 1895, was on Wednesday, on what day was the 4th July?

Section II.—Compound Addition.

160. To add compound numbers.

161. A compound number is one composed of two or more numbers of different denominations which can be reduced to the same denomination.

The sum of £6 and £4 is found by *simple* addition.

The sum of £6 12s. and £4 9s. is found by *compound* addition.

Ex. 1. Find the sum of £7 6s. 8d., £5 9s. 3d., £8 9s. 7d., and £9 7s. 9d.

£	s.	d.
7	6	8
5	9	3
8	9	7
9	7	9
29	31	27

£	s.	d.
7	6	8
5	9	3
8	9	7
9	7	9
30	13	3

We write the numbers so that units of the same name will be in the same column. Then we add the pence column as in simple addition and find the sum to be 27. Similarly with the other columns. Hence the correct sum is £29 31s. 27d. But it is usual in writing denominate numbers not to have more units of any denomination than 1 less than the number required to make 1 of the next

higher denomination; thus, a rod 12 in. long is said to be 1 ft. in length. We do not say 20 cwt. of hay, but 1 ton, etc. We therefore change the 27d. to 2s. 3d. We set down the 3d. under the pence column and add the 2s. to 31s.; 31s. + 2s. = 33s.; 33s. = £1 13s. We set down the 13s. under the shillings column and add the £1 to £29; £29 + £1 = £30.

Exercise lxxxv.

(1)

lb.	oz.	dwt.
17	9	16
25	6	12
72	11	13
57	10	19

(2)

cwt.	lb.	oz.
20	87	11
16	66	12
17	22	15
19	43	13

(3)

rd.	yd.	ft.	in.
17	4	2	6
21	2	1	7
23	3	0	8
25	5	2	9

(4)

£	s.	d.
5	5	5
8	1	$7\frac{3}{4}$
2	0	$1\frac{1}{2}$
13	0	$11\frac{3}{4}$
6	6	6

(5)

bu.	pk.	qt.	pt.
10	1	1	1
2	3	6	0
5	2	3	1
8	3	1	1
15	2	4	0

(6)

rd.	yd.	ft.	in.
37	4	1	9
30	5	2	2
	3	2	7
1	0	2	10
25	1	1	11

7. Find the sum of 1 wk. 2 da. 13 h. 40 min. 30 sec.; 2 wk. 6 da. 10 h. 8 min. 3 sec.; 5 da. 22 h. 55 min. 45 sec.; 4 h. 1 min. 15 sec.; and 1 wk. 2 da. 4 h. 5 min.

8. Add together 10 rd. 4 yd. 2 ft. 8 in.; 1 rd. 3 yd. 5 in.; 8 rd. 2 yd. 1 ft. 6 in.; 1 rd. 4 in.; and 2 yd. 1 ft. 9 in.

Section III. Compound Subtraction.

162. To subtract Compound Numbers.

Ex. 1. From 16 lb. 8 oz. 6 dwt. take 7 lb. 4 oz. 12 dwt.

lb.	oz.	dwt.
16	8	6
7	4	12
9	3	14

We write the subtrahend under the minuend, so that units of the same name will be in the same column, and begin at the right to subtract.

Since we cannot take 12 dwt. from 6

dwt., we take 1 oz. or 20 dwt. from the 8 oz., and add it to the 6 dwt., making 26 dwt. 26 dwt. – 12 dwt. = 14 dwt.

Since we took 1 oz. from 8 oz., we left only 7 oz.; 7 oz. – 4 oz. = 3 oz. 16 lb. – 7 lb. = 9 lb. Or, add 20 dwt. to the 6 dwt. and from the 26 dwt. take 12 dwt. and we have 14 dwt. Now since we added 20 dwt. to the minuend we must add an equal quantity to the subtrahend (Art. 33); we now add 20 dwt., or 1 oz., to the 4 oz. and subtracting from 8 we get 3 oz. as before.

Exercise lxxxvi.

(1)

lb.	℥	ʒ	℈	gr.
24	7	2	1	16
16	10	3	2	17

(2)

mi.	rd.	yd.
60	0	0
40	39	1

(3)

a.	sq. rd.	sq. yd.
69	25	10
10	38	15

(4)

mi.	rd.	yd.	ft.	in.
7	31	1	1	3
1	39	1	2	7

(5)

£	s.	d.
43	11	10
15	14	6

(6)

a.	sq. p.	sq. yd.
3	17	18
2	18	30

7. A farmer had 200 bu. of wheat, and sold 28 bu. 2 pk. 5 qt. 1 pt. to one man, and as much to another; how much remained?

8. A miner having 112 lb. of gold sent his mother 17 lb. 10 oz. 15 dwt. 10 gr., and 3 lb. 16 dwt. less to his father; how much did he retain?

9. From a barrel of beer containing 54 gallons, a person drew 12 gal. 3 qt. one day, and 9 gal. 2 qt. 1 pt. another; how much was left?

10. From 39 sq. rd. 29 sq. yd. 128 sq. in., subtract 17 sq. rd. 16 sq. yd. 5 sq. ft.

11. A grocer has 1 cwt. 18 lb. of sugar in one barrel, 96 lb. in another, and 1 cwt. 61 lb. in a third. After selling 1 cwt. 90 lb., how much will he have left?

12. A bicycle rider has a journey of 300 mi. to go. The first day he rides 75 mi. 48 rd.; the next 83 mi. 175 rd.; the next 68 mi. 163 rd. 5 yd. How far has he got to ride?

Section IV.—Compound Multiplication.

163. To multiply a Compound Number.

Ex. 1. Multiply 3 da. 19 hr. 59 min. by 97.

da.	hr.	min.
3	19	59
		97
291	1843	5723

da.	hr.	min.
3	19	59
		97
371	18	23

We multiply each denomination separately, as in simple multiplication, and obtain as product 291 da. 1843 hr. 5723 min. But as 5723 min. = 95 hr. 23 min., we write down 23 min., and add the 95 hr. to 1843 hr.; 1843 hr. + 95 hr. = 1938 hr. = 80 da. 18 hr., etc.

NOTE 1.—The usual method of working this example is to multiply first by 10, this product by 9, then to multiply the 3 da. 19 hr. 58 min. by 7, and add the result to the last product.

Exercise lxxxvii.

(1)			(2)				(3)			
cwt.	lb.	oz.	lb.	oz.	dwt.	gr.	da.	h.	min.	sec.
18	16	9	16	8	15	17	10	20	30	40
		5				3				7

4. What is the value of 39 oxen at £15 7s. 11½d. each?

5. What is the weight of 345 hogsheads of sugar, each weighing 14 cwt. 45 lb.?

6. What is the weight of one dozen spoons, each weighing 1 oz. 2 dwt. 16 gr.?

7. If a man owning 5 farms, of 120 ac. 52 sq. rd. each, sells 150 ac. 145 sq. rd., how much land has he left?

8. If 2 gal. 2 qt. 1 pt. leak out of a water pipe in 1 hr., what will be the waste in 1 leap year?

9. Suppose a person to walk, on an average, 3 mi. 80 rd. every morning, and 3 mi. 20 rd. 1 yd. every afternoon, how far will he walk in 2 weeks?

10. If from 2 lb. of silver enough is taken to make a dozen spoons, weighing 1 oz. 10 dwt. 2 gr. each, how much will be left?

11. What cost 97 tons of lead at £2 17s. 9½d. per ton?

12. If a man travels 17 mi. 139 rd. 3 yd. 2 ft. 7 in. in one day, how far would he travel in 38 days?

13. If 1 acre will produce 27 bu. 3 pk. 6 qt. 1 pt. of corn, what will 98 acres produce?

Section V.—Compound Division.

164. To Divide a Compound Number.

Ex. 1. Divide 80 da. 6 h. 40 min. by 17.

da.	h.	m.	da.	h.	m.
17)80	6	40	(4	17	20
68					
12 da.					
24					
294					
17					
124					
119					
5 h.					
60					
340 min.					
340					

We write the divisor at the left of the dividend. 17 is contained 4 times in 80 da. and 12 da. over; 12 da. = 288 h.; 288 h. + 6 h. = 294 h. 17 is contained 17 times in 294 h. and 5 h. over; 5 h. = 300 min.; 300 min. + 40 min. = 340 min. 17 is contained 20 times in 340 min.

Ex. 2. Divide £12 1s. 6d. by £1 6s. 10d.

$$\frac{\text{£12 1s. 6d.}}{\text{£1 6s. 10d.}} = \frac{2898\text{d.}}{322\text{d.}} = 9.$$

When we divide one compound number by another, we reduce each to the lowest denomination named in either, and divide as in simple division.

Exercise lxxxviii.

(1)

£	s.	d.
4)61	18	4

(2)

lb.	oz.	dwt.	gr.
6)76	10	14	12

(3)

t.	cwt.	lb.
7)112	16	66

4. Divide 4 gal. 2 qt. by 141.

5. Divide 40 cu. yd. 10 cu. ft. by 18.

6. Divide £48 7s. 4d. by £6 11d.

7. Divide 69 bu. 3 pk. 6 qt. by 6 bu. 3 pk. 6 qt.

8. Divide 697 lb. 7 oz. by 60 lb. 10 oz.

9. Divide 80 bu. 2 pk. 4 qt. by 13 bu. 3 pk. 5 qt.

10. A farmer put up 1000 bushels of apples in 350 barrels of uniform size; how many bushels, etc., did each barrel contain?

11. How many demijohns, each containing 2 gal. 3 qt. 1 pt. can be filled from a tank holding 71 gal. 3 qt. 1 pt. of wine?

12. A drove of cattle ate 6 t. 19 cwt. 87 lb. of hay in a week; how long will 31 t. 19 cwt. 35 lb. last them?

Section VI.—Denominate Fractions.

165. To find the value of a Fraction of a Denominate Number.

Ex. 1. How many shillings, etc., are there in $\frac{3}{8}$ of a pound?

£	s.	d.
8)3	0	0
	7	6

Since $£\frac{3}{8} = \frac{1}{8}$ of £3 (Art. 91), we divide £3 by 8 as in compound division.

Ex. 2. Find the value of $3\frac{1}{4}$ of $\frac{3}{13}$ of 2 t. 3 cwt.

$3\frac{1}{4}$ of $\frac{3}{13}$ of 2 t. 3 cwt. $= \frac{13}{4}$ of $\frac{3}{13}$ of 2 t. 3 cwt.

$= \frac{3}{4}$ of 2 t. 3 cwt.

$= \frac{6 \text{ t. } 9 \text{ cwt.}}{4} = 1$ t. 12 cwt. 25 lb.

Exercise lxxxix.

What is the value

1. Of $\frac{4}{5}$ of a bushel?
2. Of $\frac{2}{3}$ of a mile?
3. Of $\frac{1}{8}$ of a rod?
4. Of $\frac{3}{10}$ of a mile?
5. Of $\frac{7}{8}$ of a ton?
6. Of $\frac{5}{9}$ of an acre?

7. Of $\frac{3}{7}$ of £3 16s. $8\frac{3}{4}$d.? Of £18 16s. $7\frac{1}{2}$d. $\div 3\frac{2}{3}$?
8. Of $\frac{3}{5}$ of a week $+\frac{3}{4}$ of a day $+\frac{2}{3}$ of an hour?
9. Of $\frac{33}{1100}$ cwt. $-\frac{8}{13}$ of 2 lb. 8 oz.

166. To express one number as the fraction of another.

Ex. 3. Express 4 rd. 2 yd. 1 ft. 4 in. as the fraction of 1 mile.

4 rd. 2 yd. 1 ft. 4 in. = 880 in. and 1 m. = 63360 in.
Now 1 in. $= \frac{1}{63360}$ of 63360 in.;
$\therefore$ 880 in. $= \frac{880}{63360}$ of 63360 in.
Hence the fraction required is $\frac{880}{63360} = \frac{1}{72}$.

NOTE.—The example, *Express 4 lb. as the fraction of 8 lb.* may be written in any of the following ways:

1. Reduce 4 lb. to the fraction of 8 lb.
2. What fraction of 8 lb. is 4 lb.?
3. What part of 8 lb. is 4 lb.?
4. If 8 lb. is the unit, what is the measure of 4 lb.?

Exercise xc.

1. What part of an ounce is $\frac{3}{10}$ of a scruple?
2. What part of a ton is $\frac{4}{5}$ of an ounce?
3. What part of a mile is $\frac{8}{9}$ of a rod?
4. What part of an acre is $\frac{2}{3}$ of a square foot?
5. Reduce $\frac{8}{9}$ of a pint to the fraction of a gallon.
6. Reduce $\frac{5}{7}$ of an inch to the fraction of a rod.
7. Reduce $\frac{7}{8}$ of a lb. to the fraction of a ton.
8. What fraction of £3 2s. $6\frac{3}{4}$d. is 14s. $10\frac{1}{4}$d.?
9. Express 13s. $10\frac{11}{25}$d. as a fraction of £2 9s. 7d.
10. Express 2 a. 31 per. as a fraction of 4 a. 97 per.
11. Reduce $\frac{3}{2240}$ of a ton to the fraction of an ounce.
12. Reduce $\frac{1}{18}\frac{1}{720}$ of a mile to the fraction of an inch.

167. To find the value of a Decimal of a Denominate Number.

Ex. 4. What is the value of ·7875 of £1?

£·7875	·7875 of £1 = ·7875 of 20s.
20	= 15·75s.
s. 15·7500	·75 of 1s. = ·75 of 12d.
12	= 9d.
d. 9·0000	Hence ·7875 of £1 = 15s. 9d.

Ex. 5. Find the value of $2\cdot\dot{1}\dot{6}$ of 1 yd.

$2\cdot\dot{1}\dot{6}$ of 1 yd. $= \frac{23}{99}$ of 1 yd. $= \frac{1}{6}$ of 1 yd. = 2 yd. 6 in.

Exercise xci.

Find the value of

1. ·94375 of 1 acre.
2. ·815625 of 1 lb. Troy.
3. ·875 of 1s.
4. ·785 of 1 hr.
5. ·197 of 1 day.
6. ·4375 of £1.
7. ·965625 of 1 mile.
8. ·778125 of 1 ton.
9. ·628125 of £1.
10. $3\cdot458\dot{3}$ of 1s.
11. $2\cdot5381375$ of 1 day.
12. $\cdot002083\dot{3}$ of £1

168. To Express a Compound Number as a Decimal of a Higher Denomination.

Ex. 6. Reduce 17s. $5\frac{1}{4}$d. to the decimal of £1; and express £3 17s. $5\frac{1}{4}$d. in £'s only.

4	1 far.	1 far. $= \frac{1}{4}$d. = ·25d.
12	5·25d.	$5\frac{1}{4}$d. = 5·25d.
20	17·1375s.	5·25d. $= \frac{5\cdot25}{12}$s. = ·1375s.
	£·871875	$\therefore$ 17·1375s. $= \frac{17\cdot1375}{20}$£ = ·871875 £.;

Hence, £3 17s. $5\frac{1}{4}$d. = £3·871875.

Exercise xcii.

Reduce

1. 10s. 6d. to the decimal of £1.
2. 5 cwt. 64 lb. to the decimal of 1 ton.
3. 15 dwt. 15 gr. to the decimal of 1 oz. troy.
4. 248 rd. to the decimal of 1 mile.
5. 2 qt. 1 pt, to the decimal of 1 peck.
6. Express £9 5s. $4\frac{1}{2}$d. in pounds only.
7. Express 17 cwt. 89 lb. 8 oz. in cwt. only.
8. Express 7 bu. 3 pk. 1 gal. in bushels only.
9. Express $3\frac{3}{4}$ ft. as the decimal of 1 fathom.

10. What decimal of 4 oz. is 2 oz. 16 dwt. 19·2 gr.
11. Express 5 da. 9 hr. 46 min. 48 sec. in hours only.
12. Express $\frac{2}{7}$ of $\frac{1}{2}$ of $22\frac{3}{4}$ lb. as the decimal of 1 ton.

Section VII.—Practice.

169. Practice is a convenient method of solving many examples in Multiplication of Compound Numbers.

Ex. 1. Find the cost of 364 articles at $33\frac{1}{3}$ cents each.

$33\frac{1}{3}$c. = $\frac{1}{3}$ | \$364 = cost at \$1 each.
\$121.33$\frac{1}{3}$ = " $33\frac{1}{3}$c. each.

Ex. 2. Find the cost of 2 a. 3 rd. 14 per. of land at \$160 per acre.

4 | 2 × \$160 = \$320 = price of 2 a.
40 | 3 × \$40 = 120 = " 3 rd.
14 × \$1 = 14 = " 14 per.
\$454 = entire cost.

Ex. 3. Find the cost of 7 t. 6 cwt. 80 lb. of iron at \$60 per ton.

20 | 7 × \$60 = \$420 = cost of 7 t.
100 | 6 × \$3 = 18 = " 6 cwt.
80 × \$·03 = 2.40 = " 80 lb.
\$440.40 = entire cost.

Exercise xciii.

Find the price of

1.	768 articles at	25c.	5.	485 articles at	\$5.50.
2.	297 "	50c.	6.	328 "	\$1.87$\frac{1}{2}$.
3.	364 "	\$1.20.	7.	147 "	\$3.37$\frac{1}{2}$.
4.	291 "	\$1.33$\frac{1}{3}$.	8.	264 "	\$1.16$\frac{2}{3}$.

9. 15 a. 145 per. of land at \$24 per acre.
10. 9 gal. 3 qt. 1 pt. of wine at \$3.60 per gallon.
11. 84 bu. 3 pk. 1 gal. of wheat at \$1.20 per bushel.
12. 7 oz. 15 dwt. 6 gr. of gold at \$16 per ounce.
13. 20 a. 137 per. of land at \$80 per acre.
14. 3 t. 13 cwt. 40 lb. of hay at \$12 per ton.
15. What is the cost of constructing a road 17 mi. 135 rd. long at \$1880 per mile?

Exercise xciv.

Problems Involving the Previous Rules.

1. What is the value of a silver pitcher weighing 2 lb. 10 oz. avoirdupois, at $2.24 per ounce Troy?

1 oz. Troy = 480 gr.

1 lb. Avoird. = 7000 gr.

2 lb. 10 oz. " $= 2\frac{5}{8} \times 7000$ gr. $= \frac{21}{8} \times \frac{7000}{480}$ oz. Troy.

Price 1 oz. Troy = $2.24.

Price of $\frac{21}{8} \times \frac{7000}{480}$ oz. Troy $= \frac{21}{8} \times \frac{7000}{480} \times \$2.24 = \$85.75$.

2. How many pounds of gold are actually as heavy as 10 lb. of iron?

3. If a druggist buys 25 lb. Avoirdupois of drugs at $\$8\frac{1}{3}$ a pound, and sells them in prescriptions at 75 cents an ounce Apothecaries' weight, what is the gain?

4. How many sovereigns will weigh one ounce Avoirdupois, if 1869 weigh 40 pounds Troy?

5. If $\frac{3}{4}$ of an inch on a map corresponds to 7 miles of a country, what distance on the map represents 20 miles?

6. The value of 1 lb. Troy of standard gold is £46 14s. 6d.; calculate the value of a vase of the same material, whose weight is 39 oz. 18 dwt.

1 lb. = 240 dwt.; 39 oz. 18 dwt. = 798 dwt.

£46 14s. 6d. = 11214d.

Cost of 240 dwt. = 11214d.;

" 1 dwt. $= \frac{11214}{240}$d. $= \frac{5607}{120}$d.;

$\therefore$ " 798 dwt. $= \frac{798 \times 5607}{120}$d. $= \frac{133 \times 5607}{20}$d.

$= 7286\frac{1}{20}$d. = £155 7s. $2\frac{1}{20}$d.

7. If 31 cwt. of cheese cost £69 1s. 8d., what will 15 cwt. 50 lb. cost?

8. Bought 2 oz. of tea for $7\frac{3}{4}$d., what is that per lb.?

9. If 99 lb. cost £4 16s. 8d., how much is that per lb.?

10. If, when flour is $5 a barrel, the five-cent loaf of bread weighs 10 oz., what ought to be its weight when flour is $8 a barrel?

11. If $1\frac{1}{4}$ acres of land sell for $34.50, what will 20 a. 90 per. cost, at the same rate?

12. If 18 a. 140 per. cost $900, what will 150 acres cost at the same rate?

13. If $1\frac{1}{2}$ bushels of wheat cost $\$1.68\frac{3}{4}$, what will 154 bu. 1 pk. 6 qt. cost?

14. If a train travels 300 miles in 9 hr. 40 min., how long will it be in travelling 223 miles?

15. If 7 gal. 1 qt. of wine cost $17.40, what will 3 qt. 1 pt. cost at the same rate?

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16. If 15 yards, $\frac{3}{4}$ of a yard wide, will make a dress, how many yards, $\frac{5}{8}$ of a yard wide, will make another dress of the same size?

Yards required $\frac{3}{4}$ or $\frac{6}{8}$ yd. wide $= 15$ yards;
" $\frac{1}{8}$ yd. " $= 6 \times 15$ yds.;
" $\frac{5}{8}$ yd. " $= \frac{6 \times 15}{5}$ "
$= 18$ yds.

17. How many yards of cloth, $\frac{5}{8}$ yd. wide, will be required to line 35 yards, $1\frac{3}{4}$ yards wide?

18. If it requires 36 yards of carpeting, $\frac{3}{4}$ yd. wide, to cover a floor, how many yards, $\frac{7}{8}$ yd. wide, will be required to cover the same floor?

19. A regiment of 1000 men are to have new coats; each coat is to contain $2\frac{1}{2}$ yards of cloth, $1\frac{1}{4}$ yards wide, and to be lined with shalloon $\frac{3}{4}$ yd. wide; how many yards of shalloon will be required?

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20. A bankrupt owes $4000, and his assets—that is, his whole property—amount to no more than $840; what dividend will his creditors receive in the dollar?

Assets paid on $\$4000 = \$840$;
$\$1 = \$\frac{840}{4000} = \$\frac{21}{100} = 21$ cents.

21. A merchant became insolvent, owing $6850, and had only $4932 with which to pay his creditors; how much should a creditor, whose claim is $1540, receive?

22. What does a bankrupt pay in the pound if his creditors receive £376 5s. out of £2076?

23. How much will a creditor lose on a debt of $5342.25 if he receives only $67\frac{1}{2}$ cents in the dollar?

24. A creditor loses $37\frac{1}{2}$ cents in the dollar of what was due to him, and thereby loses $330; what was the sum due?

CHAPTER VII.

THE METRIC SYSTEM OF WEIGHTS AND MEASURES.

170. In 1795 France adopted a system of Weights and Measures called the Metric System, based upon the decimal system of notation, all the divisions and multiples being by 10. It has since been adopted by most of the nations of Europe and is in partial use in Canada and the United States.

171. Unit of Length. The metric standard for measurement of distance is the **Metre,** which is 39·37 inches long, or almost 3 ft. $3\frac{3}{8}$ in.

From the Metre all the other measures of this system are derived, hence the name *Metric System.*

The metre was defined by a law of the French Republic to be the distance between the ends of a rod of platinum made by Borda, the temperature being that of melting ice. It was supposed that this rod represented one ten-millionth part of the distance from the earth's equator to the pole. It has since been found that this is not exactly the case and thus the metric standard is not the terrestrial globe but Borda's platinum rod.

The annexed diagram represents one tenth of a metre in length and is called a *decimetre.* It is sub-divided into ten equal parts, each of which is called a *centimetre,* and each of these again into ten equal parts each, of which is called a *millimetre.*

172. Unit of Volume. The unit of volume depends directly on the unit of length. The capacity of a box one tenth of a metre long, wide, and deep is the standard unit of measure. Such a

NOTE.—A committee of the Imperial Parliament at the close of the Session, 1895, recommended the adoption of the Metric System through the United Kingdom, its use to be made compulsory at the end of two years.

measure is called a **Litre,** and is equal to 1·761 pints (imperial).

173. Unit of Weight. The Weight of so much distilled water at a temperature of 4°C. as would fill a measure one-hundredth of a metre (a centimetre) long, wide and deep is the standard unit of weight and is called a **Gram.**

174. By using Latin and Greek prefixes denominations higher and lower than the standard units are formed, the Greek numerals indicating multiples and the Latin ordinals indicating decimal divisions; thus:—

Deca	stands for	10	times the unit.
Hecto	"	100	
Kilo	"	1000	
Deci	"	10th.	part of the unit
Centi	"	100th.	
Milli	"	1000th.	

Metric Tables.

175. MEASURES OF LENGTH.

The unit is the Metre.

10 *millimetres (mm.)* = 1 *centimetre (cm.)*
10 centimetres - - = 1 decimetre.
10 decimetres - - = 1 *metre (m.)*
10 metres - - - - = 1 decametre.
10 decametres - - = 1 hectometre.
10 hectometres - - = 1 *kilometre (km.)*
10 kilometres - - = 1 myriametre.

The measures more commonly used are in italics.

Cloth, etc., is measured by the *metre;* very small distances by the *millimetre;* great distances by the *kilometre.*

For approximate conversions either way use the following ratios:—

8 metres = 315 inches very nearly.
8 kilometres = 5 miles nearly.

A rough rule for converting metres into English yards is to add 10 per cent. to them; thus, 40 metres are equal to 44 yards.

The centimetre is nearly equal to $\frac{4}{10}$ in., hence to convert centimetres to inches, multiply by 4 and point off the last figure.

To convert millimetres to inches or decimals of the inch: Multiply by 4 and point off the last two figures.

Exercise xcv.

A length given in any one of the measures in the foregoing table may be expressed in terms of another measure by simply moving the decimal point to the right or left: thus:

17856342 mm. = 17·856342 km.
and 4·876326 km. = 487632·6 cm.

1. How many metres in a decametre? a hectometre? a kilometre? How many decametres in a hectometre? a kilometre.

2. What part of a metre is a decimetre? a centimetre? a millimetre? What part of a centimetre is a millimetre?

3. With a metre-stick or a string one metre in length measure the height of your desk, the length and width of the schoolroom; the length and width of the platform; the width of the nearest street. Measure the foregoing with a yard measure. Convert the metric measures into feet and inches approximately and compare with actual measurement.

4. Find the value of each of the following expressions in metres:

(1) ·135 m. + 825 cm. + 1263 mm. + ·1595 km.

(2) ·927 km. − 6495 cm.; 1·37 cm. − 12·87 mm.

(3) 8 × ·0457 km.; 38019 mm. ÷ ·097.

5. Find the cost of 83·75 m. of cloth at $3.25 per metre.

6. The expense of building a certain railroad is $25000 per kilometre; what is the whole cost of the road if its length is 72 km. and 53 m.?

7. The length of the tunnel through Mont Cenis is about 12·22 km.; what is this in miles very nearly?

8. The width of the Atlantic is about 3,000 miles; what is the width in kilometres?

176. MEASURES OF SURFACE OR SQUARE MEASURE.

The unit of area is a square whose side is one centimetre. In measuring a surface we determine the number of square centimetres it contains. A table of multiples and sub-multiples of this unit is formed on the same principles of nomenclature as the tables of measures of length; but in practice these are rarely used with the exception of the

Are = 100 sq. metres, and the
Hectare = 100 ares = 10,000 sq. metres = $2\frac{1}{2}$ acres nearly.

Since the scale is 100 (10×10), there are two places for each denomination. Thus, 5 hectares, 2 ares, 5 centares, is written 5·0205 hectares, or 50205 square metres.

Exercise xcvi.

1. Write 12 square kilometres as square metres.

2. Write 8 square metres and 35 square decimetres as square metres.

3. A farmer had 5 hectares, 5 ares, 9 centares of land, and sold first ·5, then ·3 of it for $384 an are; what did he get for what he sold? How much had he left?

177. CUBIC MEASURE.

The units of this table are obtained by cubing the units of the table of Linear Measure.

1000 cu. millimetre = 1 cu. centimetre.
1000 cu. centimetres = 1 cu. decimetre = 1 litre
1000 cu. decimetres = 1 cu. metre = 1 stere.

In measuring wood the *stere* is the unit; it is about $1\frac{4}{13}$ cubic yards. $3\frac{7}{1}$ steres are about 1 cord.

Since, in cubic measure, 1000 units of each denomination make one of the next higher order, each denomination must have three places. For instance: 5 cu.m., 5 cu. dm., 5 cu. cm. is written 5·005005 cu.m.

Exercise xcvii.

1. Write 5·14 cubic decimetres as cubic metres.
2. Write 8765345 cubic centimetres as cubic metres.
3. Add 3 cu.m. 18 cu.dm. 207 cu.cm.
   385 cu.m. 230 cu.dm. 895 cu.cm. 10 cu.mm.
   831 cu.m. 300 cu.cm.

Express the sum in cu. metres, then in cu.dm.

178. LIQUID AND DRY MEASURE.

The **Litre** is the unit of the measure of capacity, and contains a volume equal to a cube whose edge is a decimetre. The multiples and sub-multiples of this unit are formed on the same principle as in the measure of length.

The *Litre* is principally used in measuring liquids, and the *hectolitre* in measuring grains, fruits, roots, etc., in large quantities.

4 litres = 7 pints very nearly.

Exercise xcviii.

1. Write in full the table of Dry and Liquid Measure.
2. Write 4 litres and 8 decalitres as litres.
3. Write 175·4 decilitres as litres.
4. At $2 a litre, what is the price of a centilitre?
5. How much must I pay for 56·25 litres of coal oil at $18\frac{3}{4}$ cents a litre?
6. How many litres in a cubic metre?
7. How many gallons in 20 litres?
8. A merchant bought 10 hectolitres of potatoes at $1 per hectolitre, and sold them at 50 cents a bushel; did he gain or lose, and how much?

179. MEASURES OF WEIGHT.

The unit is the **Gram**, which is the weight of a cubic centimetre of pure water at its greatest density.

The numeral prefixes have the same value as in linear measure. Thus:

1000 grams = 1 kilogram.

The kilogram is intended to be the weight of a litre of water at its greatest density. It is really the weight of a certain lump of platinum made by Borda in 1795 and kept at Paris.

The gram equals 15·432 Troy grains. It is used in weighing letters, mixing and compounding medicines and in weighing all light articles.

The *kilogram* equals about $2\frac{1}{5}$ lb. Avoirdupois. It is the ordinary unit of weight.

In weighing heavy articles two other weights, the *quintal* (100 kilograms) and the *tonneau or ton* (1000 kilograms), are used. The ton is a little less than 2205 lb.

The Avoirdupois ounce is about 28 grams.

Exercise xcix.

1. Write fully the table of measures of weight.

2. Write 7 kilograms and 18 grams as grams.

3. Write 247 centigrams as grams.

4. Write 12 grams, 2 centigrams and 1 milligram as grams.

5. At $6 a ton for coal, what will it cost to heat a building 30 days if it takes 400 kilograms a day?

6. How many grains in 2 grams?

7. How many pounds Avoirdupois in 976·25 grams?

8. How many grams in 12 lb. Troy? In 12 lb. Avoir.?

9. The French post-office allows 7·5 grams for a single postage; the Canadian 1 oz. Avoir. By how many grains does the Canadian exceed the French allowance?

10. The pressure of the atmosphere on a certain day was $14\frac{3}{4}$ lb. Avoir. to the square inch. What would be the corresponding pressure in kilograms to the square centimetre?

CHAPTER VIII.

PERCENTAGES.

A.—Percentages not Involving Time.

Section I.—Percentage.

180. The term *per cent.* means *by* or *on a hundred:* thus, 3 per cent. on anything means 3 on every hundred of it. Hence 1 per cent. of a number is $\frac{1}{100}$ of it; 2 per cent. is $\frac{2}{100}$ of it; 7 per cent. is $\frac{7}{100}$ of it, etc.

181. The sign, %, is generally used to represent the words per cent. Thus, 3 % is read 3 per cent.

Ex. 1. Find 5 per cent. of \$360.

Since \$100 yields \$5;

\$1 " $\$\frac{5}{100}$;

\$360 " $\$\frac{360\times 5}{100}$ or \$18.

Or, since 5 % $= \frac{5}{100} = \frac{1}{20}$; $\therefore \frac{1}{20}$ of \$360 = \$18.

Hence, we see that questions in percentage are simply applications of the Unitary Method, but in some cases may be solved more briefly by the direct application of fractions.

Exercise c.

1. A man had 50 sheep, but lost 3 of them: what part of his sheep did he lose? What per cent. of them did he lose?

$\frac{3}{50}$ or $\frac{6}{100}$ or 6 per cent.

2. A man had 25 ducks and lost 7 of them: what part of his ducks did he lose? What per cent. of them?

3. A man had \$200, and gained \$13 more; what part of his original money was his gain? What per cent. of it?

$\frac{13}{200} = \frac{6\frac{1}{2}}{100} = 6\frac{1}{2}$ per cent.

4. What per cent. of a quantity is the $\frac{1}{2}$ of it? $\frac{1}{4}$ of it? $\frac{1}{8}$ of it? $\frac{3}{4}$ of it? $\frac{1}{5}$ of it? $\frac{2}{5}$ of it?

Find

1. 16 per cent. of 450.	4. $5\frac{1}{4}$ per cent. of \$200.
2. 20 " of \$75.	5. $2\frac{1}{3}$ " of 600 men.
3. $33\frac{1}{3}$ " of 69 sheep.	6. $7\frac{1}{7}$ " of 630.

Ex. 2. A merchant sold 80 yd. of cloth from a web containing 250 yd.; what per cent. of the web did he sell?

From 250 yd. he sold 80 yd.;
1 yd. " $\frac{80}{250}$ yd.;
100 yd. " $\frac{100 \cdot 80}{250}$ yd. or 32 yd.
∴ he sold 32 %.

7. A farmer who had 800 bu. of wheat sold 320 bu.; what per cent. of his wheat did he sell?

8. A fourth of a field has been ploughed; what per cent. of the field remains to be ploughed?

9. 780 is what per cent. of 1300? of 2145?

Ex. 3. Of what number is 60, 8 %?

Since $8 = 8\ \%$ of 100;
$1 = 8\ \%$ of $\frac{100}{8}$;
$60 = 8\ \%$ of $\frac{60 \times 100}{8} = 750$.

10. Find the number of which 275 is 25 %.

11. How much must be a clerk's salary in order that 17 % of it may be \$204?

12. A farmer buys a field which adds 19 % to his original farm of 300 acres; of how many acres did the farm consist?

13. A man sold a field consisting of 25 acres, which was 12 % of his farm; how many acres were in the farm before he sold the field?

14. A farmer made a profit of 25 % by selling sheep at \$5 per head; what did he pay for the sheep?

15. A drover sold horses at \$90 a head and lost 10 %; what did he pay per head for the horses?

182. Applications of Percentage are extensive owing to the great convenience of reckoning by hundreds in business. These applications are of two classes, those not involving time and those involving time. Among those not involving

time are Commission and Brokerage, Insurance, Taxes, Profit and Loss, Commercial Discount, etc. Among those involving time are Simple Interest, Compound Interest, True Discount, Bank Discount, etc.

Section II.—Commission and Brokerage.

183. Commission is the charge made by an agent for buying or selling goods, and is generally a percentage on the *money employed in the transaction.*

184. Brokerage is the charge made by a broker for buying or selling stocks, bills of exchange, etc.

Ex. 1. My agent has bought tea, on my account, to the amount of $750; what is his commission at 2%?

The commission on $100 = $2;
" $1 = $\frac{2}{100}$;
" $750 = $\frac{750 \times 2}{100}$ = $15.

Exercise ci.

Find the commission on

1. $360 at 4%.	4. $1200 at $2\frac{1}{2}$%.
2. $790 at 2%.	5. $7600 at $3\frac{1}{4}$%.
3. $800 at $1\frac{1}{4}$%.	6. $1800 at $2\frac{1}{8}$%.

Ex. 2. I send my agent $1470 with instructions to deduct his commission at 5% and invest the balance in wheat; how much does he invest?

Commission on $100 at 5% is $5.
Sum invested out of $105 = $100;
" " $1 = $\frac{100}{105}$;
" " $1470 = $\frac{1470 \times 100}{105}$
= $1400.

7. Sent $2600 to my agent to invest after deducting his commission at 1%; what sum did he invest?

8. I sent my agent $9180 with instructions to deduct his commission at 2% and invest the balance in wheat; how much wheat did he purchase at $1.20 per bushel?

9. An agent receives \$31.55 as his compensation for purchasing goods at 4 % commission; what is the value of the goods purchased?

10. A broker sells a bill of exchange worth \$700; what is his brokerage at $\frac{1}{4}$ per cent.?

11. If a commission of \$106.47 is paid for selling \$3276 worth of goods, what is the rate per cent.?

12. An agent charges $1\frac{3}{4}$ % of the amount invested; he receives \$956.45; how much is his commission?

13. An agent who charges commission only on what he invests, received \$3672, and invests \$3600; what per cent. does he charge?

Section III.—Insurance.

185. Insurance is security guaranteed by one party on being paid a certain sum, to another against any loss.

186. The **Premium** is the sum paid for the insurance. It is always a certain per cent. of the sum insured.

187. The **Policy** is the written contract of insurance.

Ex. 1. What is the premium for insuring a house valued at \$5000 at $1\frac{1}{4}$ per cent.?

Premium on \$100 = $\$1\frac{1}{4}$;

" \$1 = $\$\frac{1\frac{1}{4}}{100}$;

" \$5000 = $\$\frac{5000 \times 1\frac{1}{4}}{100}$ = \$62.50.

Exercise cii.

Find the premium on

1. \$600 at 3 %.
2. \$840 at $1\frac{1}{3}$ %.
3. \$760 at 2 %.
4. \$375 at 3 %.
5. \$8000 at $1\frac{1}{4}$ %.
6. \$7360 at $1\frac{1}{2}$ %.
7. \$9500 at $1\frac{3}{4}$ %.
8. \$4890 at $1\frac{1}{3}$ %.

Ex. 2. For what sum should goods worth \$4900 be insured at 2 % so that, in case of loss, the owner may recover both the value of the goods and premium paid?

Premium on \$100 at 2 % is \$2.

Insurance on goods worth \$98 = \$100;

" " \$1 = \$$\frac{100}{98}$;

" " \$4900 = \$$\frac{4900 \times 100}{98}$ = \$5000.

9. For what sum must a house worth \$2400 be insured at 4 %, so that, in case it is burned, the owner may recover both its value and the premium paid?

10. What sum should be insured at 3 %, on goods worth \$6790, that the owner may receive both their value and the premium, in case of loss?

11. The premium at 2½ % on a cargo of goods amounted to \$1750; what was the value of the cargo?

12. The premium for insuring a house at ¾ % is \$24: what is the value of the house?

13. The premium for insuring a house and furniture at 1½ %, is \$79.14; what is the value of the property insured?

14. If a person who is insured for \$6000 at an annual premium of \$31.40 per \$1000, dies after 12 payments how much more will his heirs get than has been paid in premiums?

Section IV.—Taxes.

188. A Tax is a sum of money assessed on persons or property for public purposes.

1. If a man pays annually 1 % of the value of his property, estimated at \$10000, what is the amount of his tax?

2. If I am taxed 1½ % on real estate valued at \$8000, what is the amount of my tax?

3. The people of a school section wish to build a new school house, which will cost \$2850. The taxable property of the section is valued at \$190000; what will be the tax in the dollar, and what will be a man's tax whose property is valued at \$7500?

Tax on \$190000 = \$2850;

" \$1 = \$$\frac{2850}{190000}$ = \$·015 = 1½ cents;

" \$7500 = 7500 × \$·015 = \$112.50.

4. In a school section a tax of $800 is to be raised. If the amount of taxable property is $250000, what will be the tax in the dollar, and what is *A*'s tax, whose property is valued at $1800?

5. What is the assessed value of property taxed $37.50 at the rate of 5 mills on the dollar?

Value of property yielding $.005 = $1.
" " $1 = $\frac{1}{.005}$.
" " $37.50 = $\frac{37.50}{.005}$
= $7500.

6. What is the assessed value of property taxed $37.80 at the rate of $4\frac{1}{2}$ mills on the dollar?

7. A person, after paying an income tax of 22 mills on the dollar, has $2934 left; what is his income?

8. If it costs 2 % to collect, and 5 % of the tax assessed is non-collectable, what amount must be levied in order to raise $13965?

Section V.—Profit and Loss.

189. The principles of percentage, as already explained, apply to all operations for ascertaining profit and loss in business transactions.

Ex. 1. If a house costs $1200, and is sold to gain 40 %, how much is gained and what is the selling price?

1 % gain on $1200 = $12.00.
40 % " = 40 × $12.00.
= 480.
And selling price = $1200 + $480 = $1680.

1. A merchant invests $6575 in goods, and sells them to gain 18 %; what is his gain?

2. Bought 500 tons of coal at $4.20 a ton, and sold it at 27 % advance; what was the total gain and the gain per ton?

Ex. 2. I bought a horse for \$130 and sold him for \$162.50; what was my gain per cent.?

Gain on \$130 = \$32.50;

" \$1 = \$$\frac{32.50}{130}$;

" \$100 = \$$\frac{100 \times 32.50}{130}$

= \$25.

3. If I buy a pair of boots for \$6 and afterwards sell them for \$7.50, what per cent. do I gain?

4. A grocer sells a barrel of oranges for \$7.50 which cost him \$6.25; what is his gain per cent.?

5. A merchant buys sugar at 6 cents per pound and sells it at 8 cents; what per cent. does he gain?

Ex. 3. I bought calico at 12 cents a yard; for what must I sell it to gain 25 per cent.?

That for which I gave \$100 I must sell for \$125;

" " \$1 " \$$\frac{125}{100}$;

" " \$.12 " \$$\frac{.12 \times 125}{100}$

= 15 cents.

6. A merchant bought silks at \$1.25 per yard; for what must he sell them to gain 20 per cent.?

7. *A* bought a house for \$8500 and afterwards sold it at a loss of 15 %; what did he get for the house?

8. A grocer bought a quantity of sugar for \$115; for what must he sell it to gain 18 per cent.?

Ex. 4. A grocer sells a quantity of sugar for \$324, and thereby loses 10 per cent.; what did the sugar cost?

That which sold for \$90 cost \$100;

" " \$1 " \$$\frac{100}{90}$;

" " \$324 " \$$\frac{324 \times 100}{90}$

= \$360.

9. A man sells a piece of cloth for \$52.67, and thereby gains 15 per cent.; what was the cost of the cloth?

10. Sold salt at \$1.37½ per bushel, which was 5 per cent. less than cost; what was the cost?

11. The cost of coal is \$2.60, and the freight \$1.30 per ton; at what must I sell it to gain 30 %?

12. A tradesman adds 35 % to the cost price of his goods, and gives his customers a reduction of 10 % on their bills; what profit does he make?

Section VI.—Commercial or Trade Discount.

190. Commercial Discount is an allowance made by merchants and manufacturers upon their catalogue prices.

The computations in Commercial Discount are made in accordance with the principles of percentage.

Sometimes several discounts are allowed to purchasers. In such cases, the first discount is to be deducted, then the second is to be computed upon the remainder and deducted, and so on for each successive discount.

The amount of a bill less the discounts is called the **Net Amount.**

Exercise ciii.

1. An invoice was \$500, trade discounts 20 % and 5 % off; find the cost of the goods.

20 % off \$500 leaves \$400, and 5 % off \$400 leaves \$380.

The cost is \$380.

2. What is the net amount of a bill of goods, the list price of which is \$345, trade discounts 8 % and 5 % off for cash?

3. What is the difference on an invoice of \$340, between 40 % direct discount, and discounts of 25 % and 15 % ?

4. A dealer buys a book, list price \$1.00, at a discount of 20 % ; he sells the book for \$1.00; what per cent. is the profit?

5. What is the net amount of a bill of \$360, discounts being $12\frac{1}{2}$ % and 8 % ? Find a single discount equivalent to these successive discounts.

6. A man paid \$190 for goods, at discounts of 20 % and 5 % ; find the list price of the goods.

Discount of 20 % off \$100 = 20 % ;
" 5 % " \$80 = 4 %.
Both discounts = 24 % ;
76 % of list price = \$190 ;
1 % " = $\$\frac{190}{76}$;
100 % " = $\$\frac{100 \times 190}{76} = \$250.$

7. A dealer paid $288 for goods at 20 % and 10 % off; find the list price of the goods.

8. A dealer paid $320 for goods at 20 % and 20 % off; find the list price of the goods.

B.—Percentage Involving Time.

Section I.—Simple Interest.

1. If I lend you $500, and you have to pay me $1 for the use of each $100 per year, how much will I receive for 1 year?

2. How much must you pay for the use of $600 for 1 year, if you have to pay $2 for the use of each $100 per year, or 2 cents for each dollar? If you have to pay $3? $4? $8?

191. The sum paid for the *use* of money is called **Interest.**

192. The money on which the *interest* is paid is called the **Principal.**

193. The number of dollars paid for the use of $100 for 1 year is called the **Rate per cent.**

Note. When the *rate per cent.* is stated without the mention of any length of time, the time is understood to be one year.

Ex. 1. What is the interest on $2750 for 1 year at 6 per cent.?

Interest on $100 for 1 year = $6;

" $1 " = $\frac{6}{100}$;

" $2750 " = $\frac{2750 \times 6}{100}$;

= $165.

Exercise civ.

1. What is the interest on $600 for 1 year at 5 %?
2. What is the interest on $550 for 1 year at 7 %?
3. What is the interest on $3152.16 for 1 year at 7½ %?

4. A man borrowed \$7200 for 1 year, viz., \$1250 at 7 %; \$1340 at $7\frac{1}{2}$ %; \$2360 at 8 %; and the remainder at $8\frac{1}{2}$ %; how much interest has he to pay at the end of the year?

5. Four brothers have to divide equally the interest of \$25800 at 7 %; how much does each receive each year?

Ex. 2. What is the interest on \$575 for 5 years at 7 %?

Interest on \$100 for 1 year $= \$7$;

" \$1 " $= \$\frac{7}{100}$;

" \$1 for 5 years $= \$\frac{5 \times 7}{100}$;

" \$575 " $= \$\frac{575 \times 5 \times 7}{100}$

$= \$201.25.$

6 What is the interest on \$986 for 4 years at 6 %?

7. What is the interest on \$1573 for 4 years at 5 %?

8. What is the interest on \$500 for 2 years at $4\frac{1}{2}$ %?

9. What is the interest on \$2245.85 for 5 years at $6\frac{1}{2}$ %?

Ex. 3. What is the interest on \$672 for 4 yr. 8 mo. at 5 %?

4 yr. 8 mo. $= 4\frac{8}{12}$ yr. $= 4\frac{2}{3}$ years.

Interest on \$100 for 1 year $= \$5$;

" \$1 " $= \$\frac{5}{100}$;

" \$1 for $4\frac{2}{3}$ years $= \$\frac{4\frac{2}{3} \times 5}{100}$;

" \$672 " $= \$\frac{672 \times 4\frac{2}{3} \times 5}{100}$;

$= \$156.80.$

10. What is the interest on \$924 for 3 yr. 7 mo. at 6 %?

11. What is the interest on \$951 for 4 yr. 8 mo. at 7 %?

12. What is the interest on \$504.72 for 3 yr. 10 mo. at 5 %?

13. What is the interest on \$640.75 for 3 yr. 4 mo. at $4\frac{1}{2}$ %?

194. From the preceding examples we have the following rule for finding the interest on a given sum of money at a given rate per cent. for any number of years:

Multiply the Principal by the Rate per cent., the product by the number of years, and divide this result by 100.

195. The **Amount** is the name given to the sum of the principal and interest together.

Ex. 4. If a man borrows \$480 for 8 months at 8 %, what amount should he return at the end of that period?

Interest on \$480 for 12 months $= \$\frac{480 \times 8}{100}$;

" " 1 month $= \$\frac{480 \times 8}{12 \times 100}$;

" " 8 months $= \$\frac{8 \times 480 \times 8}{12 \times 100}$

$= \$25.60.$

Interest = \$ 25.60
Principal = \$480.00
Amount = \$505.60

14. What is the amount of \$840 for 10 months at 6 %?

15. What is the amount of \$1573 for 4 years at 5 %?

16. To what sum will \$784 amount in 2 yr. 9 mo. at 7 %?

In the preceding examples we have expressed the months as a fraction of a year, but in actual practice more accuracy is generally required, and we must express the given parts of a year in days.

When interest is required from one date to another, the day of the first date is to be left out, because it is not until the day following that one day's interest will have accrued.

Ex. 5. Find the interest on \$1200 from March 1, 1895, to May 31, 1898, at 7 per cent.

Time from March 1, 1895, to May 31, 1898, = 3 yr. 91 dy.

Interest on \$100 for 365 days = \$7;

" \$100 " 1 day $= \$\frac{7}{365}$;

" \$1 " 91 days $= \$\frac{91 \times 7}{100 \times 365}$;

" \$1200 " " $= \$\frac{1200 \times 91 \times 7}{100 \times 365}$;

$= \$\frac{2400 \times 91 \times 7}{100 \times 730}$

$= \$20.942...$

Interest on \$1200 for 3 yr. at 7 % = \$252.00.

" " for 3 yr. 91 days at 7 % = \$272.94.

17. Find the interest on \$500 for 156 days at 7 %.

18. Find the interest on \$7500 from May 5th to Oct. 27 at 6 %.

19. Find the interest on $8000 from Jan. 26, 1886, to March 31, 1888, at $7\frac{1}{2}$ %.

Ex. 6. At what rate per cent. must $756 be put at interest for 4 years to yield $241.92?

Interest on $756 for 1 year $= \$\frac{241.92}{4} = 60.48$;

" $1 " $= \$\frac{60.48}{756}$

" $100 " $= \$\frac{100 \times 60.48}{756}$

$= \$8$, or 8 per cent.

20. A man pays $72 for the use of $900 for 1 year, what is the rate per cent.?

21. A man lent $484 for 5 years, and received $181.50 for the interest; what was the rate per cent.?

22. If $103.68 interest is received on a principal of $432 for 4 years, what is the rate per cent.?

Ex. 7. What principal will bring $200 interest in 146 days at 5 per cent.?

Principal to give $5 in 365 days $= \$100$;

" $1 " $= \$\frac{100}{5} = \$20$;

" $200 " $= 200 \times \$20 = \$4000$;

" " in 1 day $= 365 \times \$4000$;

" " in 146 days $= \$\frac{365 \times 4000}{146}$;

$= \$10000$.

23. A man borrowed money at 7 per cent. and paid $245 interest a year; how much money did he borrow?

24. A man bequeathed his wife $875 a year, his daughter $770 a year, and his son $630 a year; what sum must be invested at 7 per cent. to produce these amounts?

25. Suppose a gentleman's interest on money, at 6 per cent., is $45 per month; how much is he worth?

Ex. 8. In what time will $800 amount to $880 at 8 per cent.?

Interest $= \$880 - \$800 = \$80$.

The interest of $800 for 1 year at 8 per cent is $64.

Time to produce $64 = 1$ year;

" " $\$1 = \frac{1}{64}$ year;

" " $\$80 = \frac{80}{64} = 1\frac{1}{4}$ years;

$=$ **1 year 3 months.**

26. How long a time would be required for $525 to gain $110.25 at 7 per cent.?

27. How long a time would it require for $625 to amount to $756.25 at 7 per cent.?

28. A principal of $600 was loaned May 20th, 1889, at $7\frac{1}{2}$ per cent. At what date did it amount to 796.87\frac{1}{2}$?

29. A note given for $273.25 at 7 per cent. remained unpaid until the interest equalled the principal. How long did it run?

Section II.—Compound Interest.

196. Compound Interest is interest, not only for the use of the sum borrowed, but also for the use of the interest if it be not paid when it falls due.

Ex. 1. What is the compound interest of $400 for 3 years at 6 per cent.?

Amt. of $100 for 1 yr. $= \$106$;

" $1 " $= \$\frac{106}{100} = \$1.06$.

" $1 for 2 yr. $= \$1.06 + $ int. of $1.06

$= \$1.06 + \frac{6}{100} \times \$1.06 = \$(1.06)^2$;

" $1 for 3 yr. $= \$(1.06)^2 + $ int. of $\$(1.06)^2$

$= \$(1.06)^2 + \frac{6}{100} \times \$(1.06)^2 = \$(1.06)^3$;

" $400 for 3 yr. $= 400 \times \$(1.06)^3 = \$476.4064$.

Amount	$=\$476.406$
Principal	$= 400.00$
Compound Interest	$= \$76.406$

Exercise cv.

1. What is the compound interest of $650 for 3 years at 6 per cent.?

2. Find the amount of $1000 for 4 years at 5 per cent.

3. Find the difference between the simple and compound interest of $350 dollars for 3 years at 8 per cent.

4. A sum of money put out at simple interest for 2 years at 8 per cent. amounted to $464; to what sum would it have amounted had it been lent at compound interest?

5. The simple interest on a sum of money for 3 years at 8 per cent. is \$120; what is the compound interest of the same sum for the same time?

6. A man deposits in the savings bank \$500, on which the interest at 4 per cent. per annum is to be added to the principal every 6 months; how much money has the man in the bank at the end of two years?

Section III.—Present Worth and True Discount.

John Smith owes me a debt of \$108 to be paid at the end of a year, without interest; how much is the debt worth at present, and how much should be allowed for the immediate payment of the debt, money being worth 8 per cent.?

If I receive \$100, and put it out to interest at 8 % for one year, it will amount to \$108; hence the present worth of the debt is \$100. Evidently \$8 should be allowed for immediate payment.

197. The **Present Worth** of a note or debt, payable at some future time, without interest, is such a sum as, being put out to interest, will amount to the given debt when it becomes due.

198. The allowance or deduction made for the payment of the debt before it becomes due is called **Discount.**

Ex. 1. What is the present worth of \$535, payable in 1 year, the rate of interest being 7 per cent.?

Amount of \$100 in 1 yr. at 7 % = \$107.

Present worth of \$107 = \$100;

" " \$1 = $\$\frac{100}{107}$;

" " \$535 = $\$\frac{535 \times 100}{107}$

= \$500.

Exercise cvi.

1. What is the present worth of \$1250.509, payable in 1 year, the rate of interest being 7 %?

2. What is the present worth of \$395.62½ payable in 1 year, when money is worth 5½ %?

Ex. 2. What is the present worth of \$787.75 due in 2 yr. 6 mo., when money is worth 6 %?

Amount of \$100 for 2 yr. 6 mo. at 6 % = \$115;
Present worth of \$115 = \$100;
" " \$1 = $\$\frac{100}{115}$;
" " \$787.75 = $\$\frac{787.75 \times 100}{115}$
= \$685.

3. What must be paid now to cancel a debt of \$994.50 due 1 yr. 9 mo. hence, at 6 %?

4. Which is the more profitable, to buy lumber at \$25 a thousand on 9 months' credit, or at \$24.50, on 6 months' credit, money being worth 6 %?

5. Bought two lots for \$2541, on 3 years' time, without interest; what is the cash value, money being worth 7 %?

6. I buy goods for \$1150 cash and sell them for \$1224 on a credit of 4 months; do I gain or lose, and how much, interest being 6 %?

Ex. 3. A note for \$1380.06 becomes due in 15 months; what deduction should be made for the immediate payment of the money, supposing money to be worth 8 %?

The interest on \$100 for 15 months = \$10;
Discount on \$110 " = \$10;
" \$1 " = $\$\frac{10}{110}$;
" \$1380.60 " = $\$\frac{1380.06 \times 10}{110}$;
= \$125.46.

7. What is the discount on \$897.82, payable in 3 years, when money is worth 7 %?

8. What is the discount on a note for \$1174.32, due in 3 yr. 3 mo., money being worth 8 %?

9. What is the difference between the interest and the discount on \$525 due in 10 mo. hence, at 6 %?

10. *A* offers *B* a farm for \$8000 cash, or for \$9000 a third cash, a third in 1 year, and a third in 2 years, the deferred payments without interest; the current rate of interest being 6 %; find the difference of the two offers.

Section IV.—Bank Discount.

A merchant desires to obtain a loan of \$1000 for 60 days, he makes a note, in the form given below, has it *indorsed* or guaranteed by some reliable person, takes it to the bank which deducts the interest on \$1000 for *63 days*, at a certain rate which varies from time to time, gives him the *proceeds*, and collects the \$1000 at the end of the 63 days.

The 3 days added to the specified time are called *days of grace*, which must elapse before payment can be *legally* enforced.

199. Bank Discount is, therefore, simple interest collected *in advance* upon the sum due on a note at its maturity.

Since bank discount is interest *in advance* upon the sum due on a note at its maturity, it will be *greater* than the *true* discount by the interest upon the true discount for the same time.

Ex. 1.

\$350.86. Toronto, July 5th, 1895.

Three months after date, for value received, I promise to pay William Robinson, or order, three hundred and fifty $\frac{86}{100}$ dollars, at the Dominion Bank.

JAMES THOMPSON.

Discounted August 1st, 1895, at 6 %.

The note *matures* nominally on Oct. 5, legally Oct. 8.

The *term of discount* is from Aug. 1, to Oct. 8 – 68 days.

The *bank discount* is the interest of \$350.86 at 6 % for 68 days, which is \$3.92.

The *proceeds* = \$350.86 – \$3.92 = \$346.94.

Exercise cvii.

1. What is the bank discount and present worth of a note of \$584, drawn Jan. 8 at 11 months, discounted at the bank May 10, at 5 % ?

2. What is the bank discount on a note for \$730 at 6 % for 30 days, days of grace included ?

3. Suppose a bill for $1200 is drawn on the 12th of August at 6 months, and paid by a banker on the 1st of January, find the money he takes off at 7 %.

Ex. 2. The proceed of a note for 30 days at 6 % are $556.92; what is the face of the note?

Bank discount for 33 days on $1 at 6 % = $.0054;
Proceeds of $1 " " = $1 − $.0054;
= $.9946;
Face of note of which proceeds is $.9946 = $1;
" " " $556.92 = $\frac{\$556.92}{.9946}$;
= $559.94.

4. I owe a bill amounting to $73.25, and I give my note for 90 days; how must I draw it to cover the discount at 7 %?

5. A. B. has a note of $1000 to pay at the Ontario Bank. At the time of its maturity he pays $300 and gives a note for 3 months, days of grace included, for the balance. The rate of discount being 8 % per annum, what was the face of the note?

6. Discounted the following note 4 months, days of grace included, before it was due at the bank of Montreal at 9 % per annum. What did I receive for it?

$500. Toronto, Jan. 1, 1895.

One year from date I promise to pay K. L. the sum of five hundred dollars with interest at 8 % per annum for value received. D. E.

7. A note for $1750 was drawn on Aug. 10 for 4 months. It was discounted on Oct. 1, at the Dominion Bank at 7½ % per annum. What sum was received for it?

8. The bank discount on a note for 3 months at 7½ % per annum is $3.45. Find the face of the note.

9. A note is drawn for 3 months, days of grace included, and when discounted at 6½ % per annum at the Ontario Bank it realizes $5.25 less than its face value. What is the face value of the note?

CHAPTER IX.

PARTNERSHIP OR PROPORTIONAL PARTS.

In this chapter it is shown how to divide numbers pure or concrete into parts which shall have to each other a given proportion.

Section I.—Sharing.

Ex. 1. Divide \$200 between A and B, so that for every \$3 that A gets, B shall get \$2.

Sum of shares $= \$3 + \$2 = \$5.$
A's share of $\$5 = \$3$;
" $\$1 = \$\frac{3}{5}$;
" $\$200 = \$\frac{200 \times 3}{5} = \$120.$

B's share may be found in a similar manner, or by simply subtracting A's share from the whole sum to be divided.

NOTE.—In the above case the shares of A and B are said to be in the ratio of 3 to 2, or in the proportion of 3 to 2.

Exercise cviii.

1. The sum of two numbers is 1260, and they are to each other as 57 and 48; what are the numbers?

2. Divide \$500 among three persons, A, B, and C, so that the three portions may be to each other as the numbers, 5, 9, and 6, respectively.

3. A bankrupt has three creditors, to whom the sums due are as the numbers 3, 4, 5; if his assets are valued at \$600, find the sums they will respectively receive.

Ex. 2. At an election the number of votes cast was 510, and $\frac{2}{3}$ of the votes for one candidate equalled $\frac{3}{4}$ of the votes for another; how many votes were cast for each?

Let A and B be the candidates.

$\frac{2}{3}$ of A's votes $= \frac{3}{4}$ of B's votes;

$\frac{1}{3}$ " $= \frac{1}{2}$ of $\frac{3}{4}$ of B's votes $= \frac{3}{8}$ of B's votes;

$\frac{3}{3}$ " $= \frac{3\times 3}{8} = \frac{9}{8}$ of B's votes;

$\frac{9}{8}$ of B's votes $+ \frac{8}{8}$ of B's votes $= 510$;

$\frac{17}{8}$ " $= 510$;

" $= \frac{8\times 510}{17} = 240$;

A's votes $= 510 - 240 = 270$.

4. A and B have 210 acres of land, and $\frac{3}{4}$ of A's share equals $\frac{6}{7}$ of B's; how many acres has each?

5. Two neighbors raised 3800 bushels of wheat, and $\frac{2}{3}$ of what one raised equalled $\frac{4}{5}$ of what the other raised; how much did each raise?

6. Two contractors have executed between them a work for which they have received \$90000; the one employed 50 laborers for 125 days of 12 hours each; the second 40 laborers during 90 days of 10 hours each; divide the money in proportion to the labor employed.

Section II. Partnership Without Time.

Ex. 1. A and B engage in trade; A furnishes \$6000 and B \$4000; they gain \$1200; what is each one's share of the gain?

The total sum in trade is \$6000 + \$4000 = \$10000;

with which they gain \$1200.

Gain on \$10000 = \$1200;

" \$1 = $\frac{\$1200}{10000} = \frac{\$12}{100}$;

" \$6000 = $\frac{\$6000\times 12}{100} = \$720 = A$'s share.

" \$4000 = $\frac{\$4000\times 12}{100} = \$480 = B$'s share.

Exercise cix.

1. A, B, and C buy a house for \$2500; A pays \$500; B \$1200; C \$800; they rent it for \$300; what is each one's share of the rent?

2. A man dying, willed to his son \$6500, to his widow \$8000, and to his daughter \$5500; but his estate amounted to only \$12000; how much did each get?

3. *A* and *B* jointly rented a pasture for \$24; *A* put in 36 cows and *B* 24 cows; how much of the rent ought each to pay?

4. *A*, *B*, and *C* hired a carriage for \$15.75, each agreeing to pay in proportion to the number of miles he rode. *A* rode 90 miles, *B* 75, and *C* 60 miles; what part of the hire ought each to pay?

5. *A* and *B* were engaged in business two years, making an annual profit of \$8190. During the first year *A* owned $\frac{2}{3}$ of the stock, and during the second year *B* owned $\frac{3}{4}$ of it; what was each partner's share of the total profit?

6. *A*, *B*, *C*, *D*, formed a partnership with a capital of \$30000. *A* furnished \$6000, *B* \$7000, *C* \$8000, and *D* the remainder. They gained 18 % of the joint stock; what was each partner's share of the profit?

Section III.—Partnership with Time.

Ex. 1. *A* and *B* engaged in trade; *A* put in \$540 for 6 months, and *B* \$450 for 8 months; they gained \$513; what was each man's share of the gain?

Gain on \$540 for 6 months = gain on $6 \times$ \$540 for 1 month.
" \$450 for 8 months = " $8 \times$ \$450 "
Total sum in trade for 1 month = \$3240 + \$3600 = \$6840.

Gain on \$6840 = \$513;
" \$1 = $\$\frac{513}{6840} = \frac{3}{40}$;
" \$3240 = $\$\frac{3240 \times 3}{40} = \$243 = A$'s gain.
" \$3600 = $\$\frac{3600 \times 3}{40} = \$270 = B$'s gain.

Exercise cx.

1. Three men *A*, *B*, and *C*, rented a pasture for \$70.56; *A* put in 36 cows for 5 months; *B* 48 cows for 4 months; and *C* 72 cows for 3 months; what part of the rent ought each to pay?

2. Bowman, Johnston, and Reed agreed to do a piece of work for $1600; Bowman furnished 7 men for 30 days; Johnston 5 men for 40 days; and Reed 6 men for 32 days; how much should each receive if they paid $95 clerk hire?

3. Two persons are in partnership 2 years; A at first put in $2500 and B $3000; at the end of nine months A took out $800 and B put in $500; they lost in two years $3825; what was each one's share of the loss?

4. A, B, and C engage in business together; A puts in $20000, and after 3 months he takes in B as a partner with $20000; at the end of 3 months more they take in C as a partner, with a capital of $10000. If they gained during the year $7800, what was each partner's share of the gain?

Section IV.—Averages.

200. The Average of several numbers is that number which substituted for each of them will produce a sum equal to that of the given numbers.

Ex. 1. Find the average of 30, 35, 42, 80 and 100.

$$30+35+42+80+100=287.$$

There are 5 numbers; therefore $\frac{1}{5}$ of 287 will be the number which substituted for each of the given numbers will produce the sum 287: $\frac{287}{5}=57{\cdot}4$.

Exercise cxi.

Find the average of

1. 16, 18, 26, 30, 36, 42, 50 and 56.
2. 17, 0, 20, 30, 70, 100, 27, 9 and 17.
3. 120, 340, 560, 780, 320 and 840.
4. Five pupils obtained the following marks at an examination, 60, 36, 75, 21, and 80, respectively; what was their average mark?
5. There were 45 pupils at school on Monday, 43 on Tuesday, 47 on Wednesday, 45 on Thursday, and 40 on Friday. What was the average attendance for the week?

6. The average temperature of the different months during the past 37 years at Toronto was, of Jan. 22°·94, Feb. 22°·58, March 29°·05, April 40°·63, May 51°·68, June 61°·84, July 67°·43, Aug. 66°·32, Sept. 58°·10, Oct. 45°·74, Nov. 36°·03, Dec. 25°·57. What was the average yearly temperature during that period?

If several values are the same, they must be repeated in making the addition, or what is the same thing, the value must be multiplied by the number of times which it occurs.

7. A grocer sold 3 lb. of tea at 75 cents per lb. and 2 lb. at 50 cents per lb.; what was the average price?

$$\frac{3\times 75 \text{ cts.} + 2\times 50 \text{ cts.}}{5} = 65 \text{ cents.}$$

8. Two purchases have been made of 15 lb. weight at $2.50 per lb., and 25 lb. at $1.80 per lb.; what is the average price per lb.?

9. A merchant sold 8 lb. of tea, 11 lb. of coffee and 25 lb. of sugar, at an average of 15 cents a pound. The tea was worth 30 cents a pound, the coffee 25 cts. a pound, and the sugar 7 cents a pound; did he gain or lose, and how much?

10. *A* mixes 25 gal. of water with 125 gal. of vinegar, which cost him 40 cents per gallon. How much must he charge per gallon so as to make a profit of $33\frac{1}{3}$ per cent.?

11. A jeweller mixes 4 oz. of gold, 18 carats fine, with 12 oz. 10 carats fine. How fine is the mixture?

12. I bought 475 barrels of flour, part at $6.50 per barrel and the rest at $6.75. The average cost per barrel was $6.\frac{12}{19}$. How many barrels at each price were bought?

13. A mixture of gold and silver weighs 15 oz. and is worth $128, but if the proportions of gold and silver be reversed it is worth $142. Gold being worth $16 per ounce, find the price of silver per ounce.

14. In a class there were six boys whose respective heights were 4 ft. 5 in., 4 ft. 6 in., 5 ft., 4 ft. $7\frac{1}{2}$ in., 4 ft. $9\frac{3}{4}$ in., and 4 ft. $1\frac{3}{4}$ in. What was the aggregate of their heights and what their average height?

CHAPTER X.

SQUARE ROOT.

1. What is the *second* power of 5? of 9? of 12?

2. What number multiplied by itself will produce 36? 49? 121? 81?

3. Find the number whose *second* power is 9; is 25; is 64; is 144.

4. Resolve each of the following numbers into *two equal* factors: 16, 25, 81, 49, 100.

201. The *Second Power* of a number is called its **Square.**

NOTE.—The square of a number is indicated by writing 2 to the right and above the given number; thus, 5^2 is read 5 squared.

202. The **Square Root** of a number is one of its *two equal factors.*

203. $1^2 = 1$. $10^2 = 100$. $100^2 = 10000$.
$9^2 = 81$. $99^2 = 9801$. $999^2 = 998001$.

Hence, it appears that if any number be divided into periods of two figures each, beginning with the units' place the number of periods will represent the number of places in the square root of the number. The left hand period may contain but one figure.

204. To find the law of the square of a number.

$25 = 20 + 5$
$25 = 20 + 5$

125	$20^2 + 20 \times 5$
50	$20 \times 5 + 5^2$
625	$20^2 + 2 \times 20 \times 5 + 5^2$

Hence the square of a number of two digits is equal to the square of the tens, plus twice the tens multiplied by the units, plus the square of the units.

Thus, instead of actually multiplying and adding we simply indicate the multiplication and addition. This we do that we may trace the tens and units separately through the process, to see where we may find them in the product.

Ex. 1. Find the square root of 2025.

$$\begin{array}{rl|l} & \dot{2}0\dot{2}5(45 & 1600+2\times40\times5+5^2(40+5 \\ & 16 & 1600 \\ \hline 85 & 425 & 2\times40+5 \mid 2\times40\times5+5^2 \\ & 425 & 2\times40\times5+5^2 \end{array}$$

Dividing the number into two periods we find that the root will consist of tens and units. We then find the largest number whose square does not exceed 20. This is 4 tens or 40. We then square the 4 tens which gives 16 hundred, and then place the 16 under the first period and subtract; this takes away the square of the tens and leaves 425, *which is twice the product of the tens multiplied by the units plus the square of the units.* Dividing 425 by twice the tens, or 80, we obtain 5, the probable units' figure of the root. Adding the units' figure (5) to the trial divisor, 80, we obtain the complete divisor, $80+5$. Since this divisor consists of twice the product of the tens plus the units, if we multiply this quantity by the number of units, we obtain twice the tens multiplied by the units, plus the square of the units, or 425, which we subtract. As there is no remainder the square root is 45.

Exercise cxii.

Find the square root of

1. 289.	5. 1296.	9. 4096.
2. 361.	6. 5625.	10. 1369.
3. 576.	7. 9025.	11. 2209.
4. 625.	8. 2401.	12. 3136.

Ex. 2. Find the square root of 4124961.

```
4|12|49|61 (2031
4
403 | 1249
      1209
4061 | 4061
       4061
```

After finding the first figure of the root and subtracting its square from the left hand period and bringing down the next period, 12, we find that 40 is not contained in 12. We therefore, put a 0 in the root, and bring down the next period. We then double the part of the root already found and write 40 as a divisor. We call it 400, and find that it goes into 1249, 3 times. We put 3 in the root and annex it to the 40. We now multiply 403 by 3 and write the product, 1209, under the 1249, and subtract, etc.

Find the square root of

13. 390625.	19. 5764801.
14. 262144.	20. 40005625.
15. 117649.	21. 25080064.
16. 499849.	22. 19228225.
17. 820836.	23. 44502241.
18. 734449.	24. 61685316.

Ex. 3. Extract the square root of ·7 to four places of decimals.

```
·70,00,00|00| (·8366
64
163 | 600
      489
1666 | 11100
       9996
16726 | 110400
        100356
          10044
```

In finding the square root of a decimal fraction care must be taken to make the decimal consist of an *even* number of figures. This is done so that the denominator of the equivalent vulgar fraction may be a complete square, which is the case in $\frac{70}{100}$, $\frac{7000}{10000}$, $\frac{700000}{1000000}$, etc. but not in $\frac{7}{10}$, $\frac{700}{1000}$, $\frac{70000}{100000}$, etc

Find the square root of

25. .2209.	30. .714.	35. 71.
26. .0729.	31. .895.	36. 6.4.
27. .1024.	32. .9.	37. 14.4.
28. .0004.	33. 16.9.	38. 50.1264.
29. .005	34. 3.61.	39. 5416.96.

Exercise cxiii.

1. A man owns a farm in the form of a square, which contains 10 acres; how many rods in length or breadth is it?

2. What would it cost to fence a square lot containing 160 acres, at the rate of $4 per rod?

3. I have a room in the form of a square, which requires 100 yards of carpet to cover it; what is the size of the room, if the carpet is 1 yard wide?

4. What is one of the equal factors of 15625.

5. A garden contains 1452 square yards, and it is 3 times as long as it is broad. Find its length and breadth.

6. A certain public hall in a city contains 2646 square feet, and its length is $1\frac{1}{2}$ times its breadth; what are the dimensions of the hall?

7. A merchant bought a certain number of yards of muslin, giving as many cents for each yard as there were yards. The whole cost $72.25. How many yards did he buy and at what price per yard?

8. What must be the dimensions in feet and inches of a square garden-lot, which shall be equal to two rectangular ones measuring respectively 8 by 10 and 8 by 18 rods?

9. A rectangular court that is twice as long as it is wide contains 31250 square feet. How long and wide is it?

10. A body of soldiers in column forms 361 ranks 9 abreast; if they were drawn up in solid square, how many would there be in each face?

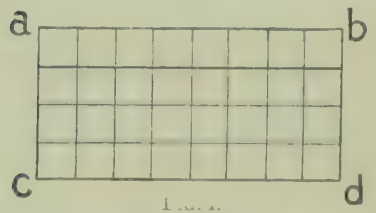
11. A piece of cloth is 5 times as long as broad, and costs £19; supposing the price to be 4s. 9d. a square yard, find the dimensions of the piece.

12. The area of a square field is 10 acres: what will it cost to build a wall round it at 85 cents per square yard of walling, if the wall be 2 yards high?

CHAPTER XI.

MEASUREMENT OF SURFACES AND SOLIDS.

Section I.—The Rectangle.



205. **a c d b** has its sides parallel and its angles right angles. It is called a Rectangle.

Suppose the line **a b** to be divided into 8 units, and **a c** into 4 of the same units; draw lines through the points of division parallel to **a c**, **a b** respectively. Then the rectangle is divided into a number of equal squares, each constructed on a line equal to the unit of linear measurement.

Evidently their number may be found by multiplying the number of square units of measure which can be placed on one side of the rectangle by the number of corresponding linear units in the adjacent side.

It is of the utmost importance that this first principle of surface measurement be thoroughly understood, as the more difficult ones will be traced back to this fundamental principle.

Ex. 1. A room is 16 feet long and 15 feet wide; what is its area?

Area of surface 1 ft. long by 1 ft. wide = 1 sq. ft.
" 18 ft. long by 1 ft. wide = 18 sq. ft.
" 18 ft. long by 15 ft. wide = 15 × 18 sq. ft.
= 270 sq. ft.

Exercise cxiv.

Find the area of the rectangles having the following dimensions:

1. 8 ft. by 12 ft.
2. $6\frac{1}{2}$ ft. by 14 ft.
3. 21 ft. by 25 ft.
4. 2 yd. 2 ft. by 7 yd.
5. 17 yd. by 20 yd. 2 ft.
6. 19 ft. 7 in. by 24 ft.

7. What will it cost to plaster a room 16 ft. long, 14 ft. wide and 9 ft. high, at 17 cents a square yard?

8. How many acres in a rectanglar field 30 rods long and 22 rods wide?

9. How many bricks 8 in. long and 4 in. wide are required to make a walk 78 ft. long and 4 ft. wide?

10. Find the surface of a pane of glass measuring $37\frac{1}{2}$ in. long and 23 in. wide.

Section II.—Carpeting Rooms.

206. Carpets are sold in strips, and when the width of a strip is known, we can ascertain what *length* of carpet will be required to cover a given surface.

Ex. 1. How many yards of carpet 2 ft. 3 in. wide will be required for a room 21 ft. by 18 ft.?

Area of surface to be covered . . . $= 18 \times 21$ sq. ft.
Length of carpet 1 ft. wide, required to cover given area . . . . . $= 18 \times 21$ feet.
Length of carpet, $2\frac{1}{4}$ ft. wide, required to cover given area . . . . . $= \frac{18 \times 21}{2\frac{1}{4}}$ ft.
$= 56$ yards.

Exercise cxv.

How many yards of carpet 27 in. wide will be required for rooms whose dimensions are

1. 27 ft. by 21 ft.?
2. 15 ft. by 12 ft.?
3. 18 ft. by 24 ft.?
4. 26 ft. by 36 ft.?

Find the cost of carpeting rooms whose dimensions are

5. 18 ft. by 20 ft., with carpet 3 ft. wide, at \$1.20 a yd.
6. 20 ft. by 24 ft., with carpet 30 in. wide, at 90 cts. a yd.
7. 15 ft. by $17\frac{1}{2}$ ft., with carpet 3 ft. wide, at \$1 a yd.

Section III.—Papering a Room.

207. Room papers, like carpets, are sold in strips, and we ascertain the quantity that will cover a wall in the same manner as we ascertained the quantity of carpet required to cover a floor.

Ex. 1. How many yards of paper 16 in. wide will be required for a room 18 ft. long, 14 ft. wide, and 8 ft. high, which contains 1 door 7 ft. high by $3\frac{1}{2}$ ft. wide and 3 windows each 5 ft. high by $2\frac{1}{2}$ ft. wide?

Length of surface to be covered . . . . . $= (18 + 14 + 18 + 14)$ ft. $= 64$ ft.
Area of entire walls . . . $= (8 \times 64)$ sq. ft. $= 512$ sq. ft.
Area of door . . . . . $= (3\frac{1}{2} \times 7)$ sq. ft. $= 24\frac{1}{2}$ sq. ft.
Area of 3 windows . . . $= (3 \times 2\frac{1}{2} \times 5)$ sq. ft. $= 37\frac{1}{2}$ sq. ft.
Area of door and windows $= (24\frac{1}{2} + 37\frac{1}{2})$ sq. ft. $= 62$ sq. ft.
Area to be papered . . . $= (512 - 62)$ sq. ft. $= 450$ sq. ft.
450 square feet . . . . $= 450 \times 144$ sq. in.
∴ length of paper required $= \frac{450 \times 144}{16} = 4050$ in.
$= 112\frac{1}{2}$ yards.

Exercise cxvi.

1. How many yards of paper 20 in. wide will be required for a room 20 ft. long, 15 ft. wide, and 9 ft. high?

2. How many sq. ft. of paper will be required for a room 18 ft. 9 in. long, 15 ft. 3 in. wide, and $8\frac{1}{2}$ ft. high?

3. A room 24 ft. long, 20 ft. wide, and 10 ft. high contains 2 doors each 7 ft. by 4 ft., and 6 windows each $5\frac{1}{2}$ ft. by 4 ft.; find how many yards of paper 2 ft. wide will be required to paper it?

4. How many yards of paper 30 in. wide will it require to cover the walls of a room 15 ft. long, 12 ft. wide, and 8 ft. high?

5. William Benson has agreed to plaster the walls and ceiling of the room in the last example, at 10 cents per sq. yd.; what will his bill amount to?

Section IV.—The Parallelogram.

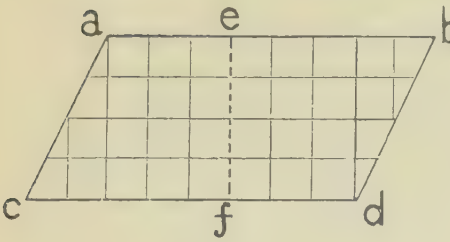


FIG. 2.

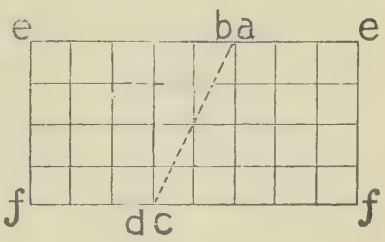


FIG. 3.

208. A **Parallelogram** is a four-sided figure whose opposite sides are parallel.

Make a parallelogram **a c d b,** Fig. 2, from a piece of board about half an inch thick. Divide vertically as in **e f** and insert pins so that it may be easily put together again. Change the parallelogram into a rectangle as in Fig. 3. The area of the rectangle is found as in Art. 205. Its length is the length of the parallelogram and its breadth is the perpendicular height of the parallelogram.

It is evident that into whatever form any block may be changed, its surface must remain the same.

Exercise cxvii.

1. A county ditch 18 ft. wide and 105 rods long was dug diagonally across a farm; how many square rods did the owner lose?

2. A farmer buys a farm of 100 acres at $45 per acre, across which a river runs diagonally. If the river measures 87 ft. wide and 62 rods long, how much is the land worth that it covers?

3. How many rectangles, each having a base 12 ft. long and a breadth of 8 ft., would be required to have the same area as a parallelogram 48 ft. long with a perpendicular height of 12 ft.?

4. If a parallelogram 16 ft. long and 12 ft. wide were changed into the form of a rectangle, what would be the area of the rectangle?

Section V.—The Triangle.

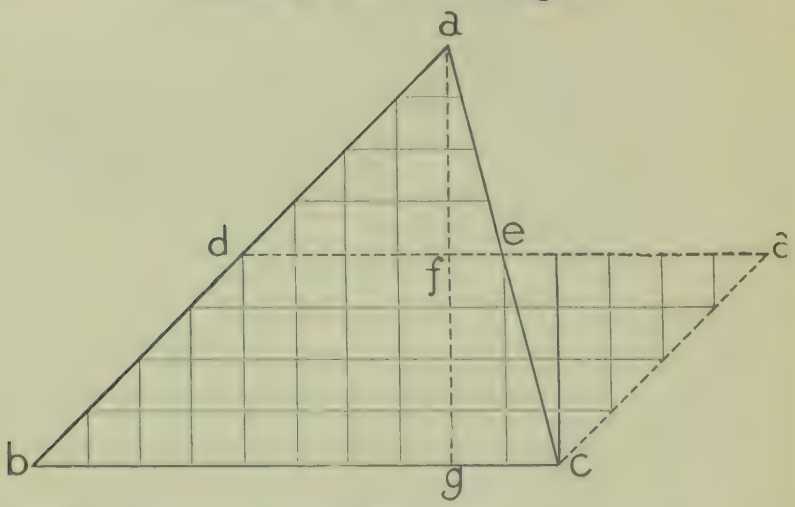


FIG. 4.

209. Make from a board a triangle **a b c**, whose sides are bisected in **d** and **e**. Turn the triangle round **e** into the position **a e c**, thus changing the triangle **a b c** into the parallelogram **a c b d**, whose length is the base of the triangle and whose width, **f g**, is half the perpendicular height. If **b c e d** be cut vertically in **f g** the triangle may be changed into a rectangle, as in Art. 208.

A mental picture is thus formed in the pupil's mind of the fact that any triangle can be changed into a parallelogram or rectangle of the same area, the same base and half the perpendicular height.

Exercise cxviii.

1. The gable end of a barn measures along the base 24 ft. and the perpendicular distance from the base to the ridge is 12 ft. How many square feet of lumber will be required to inclose it?

2. A rectangle is 16 inches long and 10 inches wide, and a triangle has a base 8 inches long and a perpendicular height of 10 inches; how many times greater is the rectangle than the triangle?

3. How much will it cost to inclose a barn which is 36 ft. long, 24 ft. wide, 14 ft. to the plates, and 10 ft. from the plates to the ridge, with lumber at $14 per thousand?

4. The slant height of an octagonal steeple is 56 ft. and the circumference of its base is 32 ft.; how much will it cost to cover it with sheets of tin 28 by 10 in. at 7 cents per sheet?

5. The area of a triangular field is $1\frac{1}{2}$ acres, and the perpendicular height is 750 links; find the length of the base.

Section VI.—The Right-angled Triangle.

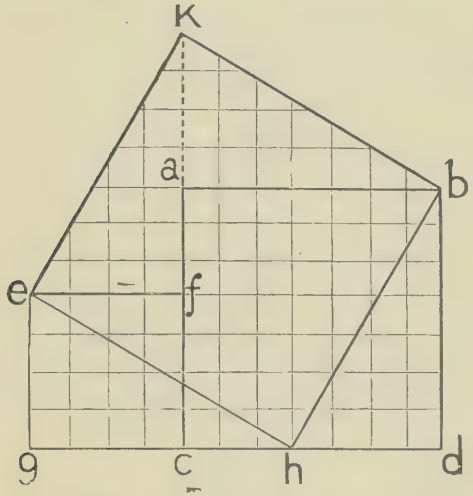


Fig. 5.

210. In Fig. 5 take **h d = g c.** Then the square on **g c** is equal to the square on **h d.** **b h d** is a right-angled triangle and the squares **a b d c, e f c g** are the squares on its sides. Move each of the triangles **h e g, b h d** along the hypothenuse of the other, without rotation into the positions **a b k, e f k.** The figure formed is the square on **b h,** the hypothenuse of the right-angled triangle **b h d,** and is evidently equal to the sum of the squares on its sides.

Hence in any right-angled triangle the square on the hypothenuse is equal to the sum of the squares on its sides.

Exercise cxix.

1. If the gable end of a house 34 ft. wide is 14 ft. high, what is the length of the rafters?

2. Find the width of a house whose rafters are 12 ft. and 15 ft. respectively and form a right angle where they meet.

3. A ladder 78 ft. long stands close against a building; how far must it be drawn out at the foot that the top may be lowered 6 ft.?

4. A tree was broken 51 ft. from the top, and fell so that the end struck 24 ft. from the foot; what was the length of the tree?

5. What is the shortest distance between the lower corner and the upper opposite corner of a room 60 ft. long, 32 ft. wide, and 51 ft. high?

Section VII.—The Circle.

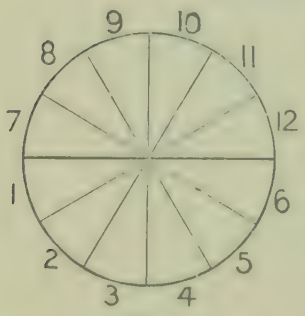


FIG. 6.

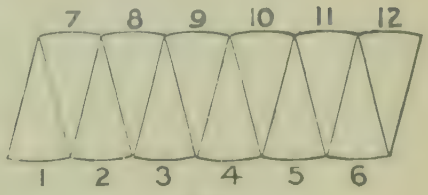


FIG. 7.

211. From a piece of sole leather, make a circle and divide it into triangles as in Fig. 6. By changing the circle into the form of Fig. 7 we have a parallelogram. The area of the parallelogram is equal to the length multiplied by the breadth. The length is evidently half the circumference and the breadth is half the diameter, which is the radius of the circle.

Hence, to find the area of a circle, multiply half the circumference by the radius.

It is found that the circumference of a circle divided by the diameter = 3·1416, which is usually denoted by the Greek letter π.

Hence $\frac{c}{2r} = \pi$; $\therefore\ c = 2\pi r$.

Again, area $= \frac{1}{2}c \times r = \frac{1}{2} \times 2\pi r \times r = \pi r^2$.

Exercise cxx.

1. How many yards of carpeting 1 yd. wide would be required to cover a circular hall 15 ft. in diameter?

2. The circumference of a race-track is one mile; what is the area?

3. How many feet of iron must a blacksmith buy for the tires of 12 wheels 4 ft. in diameter?

4. What space would the driving wheel of a locomotive 5 ft. 3 in. in diameter pass over in making 25 revolutions?

5. How much farther will a horse have to run in going round the sides of a square mile of land, than in going round the same area in the form of a circle?

Section VIII.—Rectangular Solids.

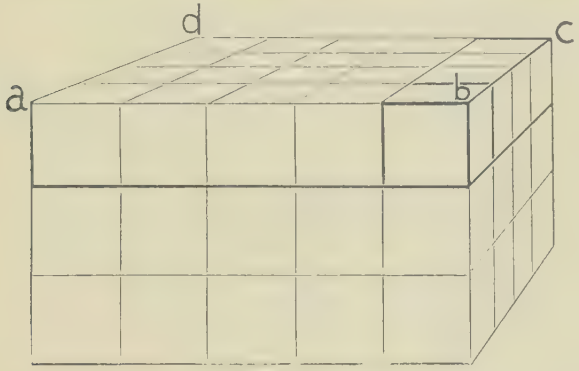


FIG. 8.

Fig. 8 is a Rectangular Solid of which **a b c d** is its upper surface. As in the rectangle, Art. 205, lines are drawn dividing the sides into equal squares. The number of cubic units of measure which can be placed on the line **b c** corresponds to the number of linear units of measure in

that line; and the number that can be placed on the surface **a b c d** is the number on **b c** multiplied by the number of linear units of measure in the adjacent side **a b.** If the number of cubic units of measure on **a b c d** be multiplied by the number of linear units of measure corresponding to the height of the solid the product will evidently be the number of cubic units of measure in the whole solid.

Hence, to find the number of cubic units of measure in a rectangular solid, *multiply the number of cubic units of measure that can be placed on a line called the breadth, by the number of linear units of measure in the length, and this product by the number of linear units of measure in the height.*

Ex. 1. Find the number of cubic feet in a room 16 ft. long, 14 ft. wide and 9 ft. high.

No. of cu. ft. which can be placed on length = 16 cu. ft.
" " " upper surface = 14×16 cu. ft.
Whole number of cubic feet in room . = $9 \times 14 \times 16$ cu. ft.
= 2016 cu. ft.

Exercise cxxi.

Find the cubic content of the rectangular solids whose dimensions are

1. 8 ft., 6 ft., 5 ft.
2. $2\frac{1}{2}$ ft., $5\frac{1}{5}$ ft., $7\frac{1}{7}$ ft.
3. 3 ft., $7\frac{1}{2}$ ft., $8\frac{1}{3}$ ft.
4. 2·6 ft., 3·5 ft., 5 ft.

Ex. 2. How many bricks will be required to build a wall 20 ft. long, 15 ft. high, and 18 in. thick, each brick being 8 in. long, 4 in. wide, and 3 in. thick?

Cubic content of wall . . = $(20 \times 12 \times 15 \times 12 \times 18)$ cu. in.
" " brick . . = $(8 \times 4 \times 3)$ cu. in.
∴ number of bricks required = $\frac{20 \times 12 \times 15 \times 12 \times 18}{8 \times 4 \times 3}$
= 8100

5. How many bricks will be required to build a wall 15 ft. long, 20 ft. high and 15 in. thick, each brick being 9 in. long, $4\frac{1}{2}$ in. wide and 3 in. thick?

6. What will it cost to put a stone foundation under a barn 36 ft. long, by 24 ft. wide at $2 a cubic yard, the wall being 7 ft. high and 2 ft. thick?

7. The length and breadth of a tank are 60 and 42 ft. respectively; find the depth in order that it may contain 840 cubic yards.

8. The breadth and thickness of a beam are 20 and 15 inches respectively; find the length of a piece which contains 10 cubic feet.

Ex. 3. How many cords of wood in a pile 36 ft. long, 10 ft. 6 in. wide and 4 ft. high?

No. cu. ft. which can be placed on length = 36 cu. ft.
" " " " upper surface = $10\frac{1}{2} \times 36$ cu. ft.
" " in pile . . . . . = $4 \times 10\frac{1}{2} \times 36$ cu. ft.
" " in cord . . . . . = 128 cu. ft.
" cords in pile . . . . . = $\frac{4 \times 10\frac{1}{2} \times 36}{128}$ cords
= 11·8125 "

9. How many cords of wood in a pile 50 ft. long, 11 ft. 3 in. high, and 5 ft. wide?

10. How many cubic inches in a tank 9 ft. 6 in. long, 5 ft. 3 in. wide, and $3\frac{1}{3}$ ft. deep? How many Imperial or standard gallons will it hold?

11. We receive the oil used in our lamp in a tin can 8 in. long and wide, and 12 in. deep; how many gallons in the can when full?

12. Find the weight of 6 planks, each 12 ft. 6 in. long, $11\frac{1}{4}$ in. wide, and 4 in. thick, if the weight of a cubic foot of the wood is 24 lb.

13. A piece of copper 1 ft. long, 9 in. wide, and $\frac{5}{8}$ in. thick is rolled into a plate 6 ft. long and 4 ft. wide; how thick will the plate be?

14. A log of square timber is 18 ft. long, 18 in. broad, and 15 in. thick; if $2\frac{1}{2}$ solid feet are cut off the end of it, what length is left?

15. A cistern is partly full of water and has 250 gallons in it. If the length of the cistern be $6\frac{1}{4}$ ft. and the breadth $3\frac{1}{2}$ ft., find the depth of the water.

16. A cistern is $12\frac{1}{2}$ ft. long and 8 ft. wide; if when it is full of water 1500 gallons are drawn off, how much will the surface of the water sink?

CHAPTER XII.

Miscellaneous Problems.

1. A garrison of 800 men had provisions to last for 60 days, but 15 days afterwards 80 men were killed; how long will they last the remainder?

They would last 800 men 15 days.

" 1 man 800×45 days.

" 720 men $\frac{800 \times 45}{720}$ days $= 50$ days.

2. 28 shanty men have provisions for 20 days, but 7 men more arrived; how long will the provision now last?

3. A garrison of 1000 men were victualled for 28 days; after 11 days it was reinforced by 2400 men; how long will the provisions last?

4. A garrison of 150 men had provisions for 5 months, but 200 men were sent away; how long will the provisions last the remainder?

5. A garrison of 1000 men were victualled for 30 days; after 10 days it was reinforced by 3000 men; in what time would the provisions be exhausted?

6. A can do a piece of work in 8 days, and B can do it in 9 days; how long will it require A and B working together to do it?

The part A does daily . $= \frac{1}{8}$

" B " " . $= \frac{1}{9}$

" A and B do daily $= \frac{1}{8} + \frac{1}{9} = \frac{17}{72}$

$\therefore$ they do $\frac{17}{72}$ in $\frac{1}{17}$ day;

$\therefore$ they do the whole work in $\frac{72}{17}$ days, or $4\frac{4}{17}$ days.

7. A can do a piece of work in 12 hrs., and B can do it in 15 hrs.; in what time can both working together do the work?

8. A can do a piece of work in 20 days, B can do it in 24 days, and C can do it is 30 days; in what time will they all do it working together?

9. A can build a wall in 8 days, B in 12 days, and C in 15 days; in what time can they all build it working together?

10. A quantity of flour lasts a man and wife 9 days, and the wife alone 27 days; how long would it last the man alone?

11. A can do a piece of work in 20 days; after working at it for 8 days B comes to help him and they finish the work in 5 days; how long would it take B by himself to do the work?

12. A can do $\frac{2}{3}$ of a piece of work in 8 days; B can do $\frac{3}{4}$ of the same work in 12 days; in what time could both working together do 2 such pieces of work?

13. A and B can mow a field in 12 days; A and C in 15 days; B and C in 20 days; in what time could A mow it by himself?

A and B can do $\frac{1}{12}$ of work in 1 day;
A and C " $\frac{1}{15}$ "
B and C " $\frac{1}{20}$ "
$\therefore$ 2 A's and 2 B's and 2 C's " $\frac{1}{12}+\frac{1}{15}+\frac{1}{20}$ " $=\frac{1}{5}$;
$\therefore$ A and B and C " $\frac{1}{2}$ of $\frac{1}{5}$ " $=\frac{1}{10}$;
$\therefore$ A " $(\frac{1}{10}-\frac{1}{20})$ " $=\frac{1}{20}$;
$\therefore$ A can do the work in 20 days.

14. A and B can do a piece of work in 8 days; A and C can do it in 9 days, and B and C in 10 days; in what time can all three working together do it?

15. A and C can dig a garden in 10 days: B and C can dig $\frac{4}{5}$ of the same garden in 4 days, and B alone can dig it in 20 days; in what time can A do it by himself?

16. A piece of work has been half done by A, B, and C working together, in 8 days: if A and B together can finish it in 12 days, in what time could C have finished it?

17. *A* can do a piece of work in 6 days of 10 hours each, and *B* can do it in 8 days of 9 hours each; for how many hours a day should *A* and *B* be engaged together, that the work may be done in 4 days?

18. If 6 men or 9 women can do a piece of work in 12 days, in what time will 4 men and 7 women do it?

6 men do the work in 12 days ∴ 1 man does $\frac{1}{72}$ of it in 1 day.
9 women do the work in 12 days ∴ 1 woman does $\frac{1}{108}$ of it in 1 day;
∴ 4 men and 7 women do $\frac{4}{72} + \frac{7}{108}$ or $\frac{13}{108}$ of it in 1 day;
∴ " " do $\frac{1}{108}$ of it in $\frac{1}{13}$ day;
∴ " " do it in $\frac{108}{13}$ day, or $8\frac{4}{13}$ days.

19. If 7 boys or 4 men can do a piece of work in 9 days, in what time can 4 boys and 7 men do it?

20. If 3 men or 5 women do a piece of work in 12 days, in what time can 2 men and 1 woman do it?

21. If 1 man and 2 women can do a piece of work in 8 days, and 3 men and 4 women can do it in 3 days, in what time can 1 man or 1 woman do it?

Since 1 man and 2 women do $\frac{1}{8}$ of it in 1 day;
∴ 2 men and 4 women do $\frac{1}{4}$ " "
But 3 men and 4 women do $\frac{1}{3}$ " "
∴ 1 man does $\frac{1}{3} - \frac{1}{4}$, or $\frac{1}{12}$ " "
∴ 1 man will do it in 12 days.
Now 1 man and 2 women do $\frac{1}{8}$ of it in 1 day;
∴ 2 women do $\frac{1}{8} - \frac{1}{12}$, or $\frac{1}{24}$ " "
∴ 1 woman will do it in 48 days.

22. If 3 men and 2 boys do a piece of work in 8 days, and 3 men and 7 boys can do it in 6 days, in what time can 1 man or 1 boy do it?

23. If 2 men and 5 boys can do a piece of work in 20 days, and 1 man and 8 boys can do it in 18 days, in what time can 1 man or 1 boy do it?

24. If 7 men and 5 women can do a piece of work in $2\frac{2}{3}$ days, and 3 men and 8 women can do it in $3\frac{1}{13}$ days, in what time can 1 man or 1 woman do the work?

25. 3 women and 2 boys can do a work in $6\frac{2}{3}$ days, and 2 women and 3 boys can do it in $7\frac{1}{17}$ days; in what time can 1 woman or 1 boy do it?

26. A cistern is filled by 2 pipes in 8 and 10 hours respectively; in what time will they fill it when they both run at the same time?

They fill $\frac{1}{8} + \frac{1}{10}$ of the vessel in 1 hour;
∴ " $\frac{5+4}{40}$, or $\frac{9}{40}$ " "
∴ " $\frac{1}{40}$ in $\frac{1}{9}$ hour;
∴ they fill the vessel in $\frac{40}{9}$ or $4\frac{4}{9}$ hours.

27. A vessel is filled by 3 taps, running separately, in 60, 75 and 90 minutes respectively; in what time will they fill it when they all run at the same time?

28. Two pipes running together can empty a cistern in 8 hours, and one by itself can do it in 12 hours; in what time can the other empty it?

29. Two pipes running together can empty a vessel in 50 minutes; one of them can empty $\frac{2}{5}$ of the vessel in 40 minutes; in what time can the other empty $\frac{1}{3}$ of it?

30. A cistern is filled by two pipes, *A* and *B*, in 20 and 24 minutes respectively, and is emptied by a tap, *C*, in 30 minutes; in what time will it be filled by all running together?

31. A bath is filled by a pipe in 60 minutes; it is emptied by a waste pipe in 40 minutes; in what time will the bath be emptied if both pipes are opened at once?

One pipe empties $\frac{1}{40}$ of vessel in 1 minute.
The other fills $\frac{1}{60}$ of vessel in 1 minute.
∴ when both are running $(\frac{1}{40} - \frac{1}{60})$, or $\frac{1}{120}$ of the vessel is emptied in 1 minute.
∴ the vessel is emptied in 120 minutes.

32. A vessel can be filled by 2 taps running separately in 30 and 36 minutes respectively, and emptied by a third in 15 minutes; if the vessel is full and all 3 taps running at once, in what time will it be emptied?

33. A bath can be filled by two taps running separately in 20 and 30 minutes respectively, and emptied by two others in 24 and 18 minutes respectively; if the bath is full and all four taps opened, in what time will the bath be emptied?

34. *A* gave to James $\frac{1}{3}$ of his money and to John $\frac{5}{18}$ of it, and had \$2.10 left; how much had he at first?

He gave away $\frac{1}{3} + \frac{5}{18}$, or $\frac{11}{18}$ of his money.
He had left $\frac{18}{18} - \frac{11}{18}$, or $\frac{7}{18}$ "
$\therefore \frac{7}{18}$ of his money = \$2.10;
$\frac{1}{18}$ " = $\$\frac{2.10}{7}$;
$\frac{18}{18}$, or his money = $\$\frac{18 \times 2.10}{7}$
= \$5.40.

35. A father willed to his eldest son $\frac{2}{5}$ of his property; to his second son $\frac{2}{7}$ of it, and to his youngest son the rest, amounting to \$7238. What was the property worth?

36. A post is $\frac{1}{6}$ in the earth, $\frac{3}{7}$ in the water, and 13 feet above the water. What is the length of the post?

37. A man devotes ·12 of his income to charity, ·25 for educating his children, ·15 for household expenses, and saves the remainder, which is \$284.76. What is his income?

38. A ship whose cargo was worth \$25000 being disabled, ·45$\frac{1}{2}$ of the whole cargo was thrown overboard. What would a merchant lose who owned ·25 of the cargo?

39. A laborer in one week dug 5 rods more than $\frac{1}{2}$ the length of a ditch, and the next week he dug the remaining 20 rods; how long was the ditch?

Length of ditch dug first week = $\frac{1}{2}$ ditch + 5 rods;
$\therefore$ length remaining . . . = $\frac{1}{2}$ ditch less 5 rods;
$\therefore \frac{1}{2}$ length of ditch less 5 rods = 20 rods;
$\therefore \frac{1}{2}$ length of ditch . . . = 20 rods + 5 rods;
= 25 rods;
$\therefore$ length of ditch . . . . = 50 rods.

40. A man invested \$800 more than $\frac{2}{3}$ of his money in a house, and \$600 more than $\frac{1}{4}$ of the remainder in a lot, and had now \$900 left; how much was he worth?

41. If 10 men can chop 90 cords of wood in 8 days, how many cords can be chopped by 20 men in 4 days?

Cords chopped by 10 men in 8 days = 90 cords;
" 1 *man* in 8 *days* = $\frac{90}{10}$ = 9 *cords;*
" 1 man in 1 day = $\frac{9}{8}$ cords;
" 20 *men* in 1 *day* = $\frac{20 \times 9}{8} = \frac{45}{2}$ *cords;*
" 20 men in 4 days = $\frac{4 \times 45}{2}$ = 90 cords.

NOTE.—When the pupil has become familiar with the unitary system, and thoroughly understands the reason of each step, the process may be abridged by leaving out the steps in italics.

42. If 8 men build 33 feet of wall in 11 days, in how many days will 12 men build 36 feet?

43. If 36 men earn $324 in 18 days, how much will 42 men earn in 87 days?

44. How many days will it take 15 men to cut 810 cords of wood, working 9 hours a day, if 13 men can cut 364 cords in 15 days, working 12 hours a day?

45. It costs a family of 5 persons $135 for 6 weeks' board? how much will it cost a family of 7 persons at the same rate for 3 weeks?

46. If 12 men can dig a ditch 16 rods long in 8 days, in how many days can 24 men dig a ditch of the same depth and width, 32 rods in length?

Time in which 12 men will dig 16 rods = 8 days;
" 1 man " 1 rod = $\frac{12 \times 8}{16}$ days;
" 24 men " 32 rods = $\frac{32 \times 12 \times 8}{24 \times 16}$ days;
= 8 days.

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47. At what time between 5 and 6 o'clock will the hour and minute hands of a clock be together?

In 12 hours the minute hand passes the hour hand 11 times. If both hands are together at 12 o'clock:

Time in which min. hand will 1st time pass hour hand . . . . . = $\frac{1}{11}$ of 12 hrs.

Time in which min. hand will 5th time pass hour hand . . . . . = $\frac{5}{11}$ of 12 hrs.
= 5 h. 27 $\frac{3}{11}$ min.

or 27$\frac{3}{11}$ min. past 5 o'clock.

48. At what time between 7 and 8 o'clock are hour and minute hand of a clock together?

49. A gentleman being asked the time of day, replied that it was between 3 and 4 o'clock, and the hour and minute hands were exactly together; what was the time?

50. At what time between 2 and 3 o'clock are the hands of a clock at right angles to each other?

When the hands are at right angles there are 15 minute-divisions between them.

Since 2 o'clock there are 10 minute-divisions between the hands. It is required, therefore, to find how long it will take the minute hand to gain 10 + 15 or 25 minute-divisions on the hour hand.

The minute hand gains 55 minute-divisions in 60 minutes;
∴ time required by min. hand to gain 11 min.-div. = 12 min.

" " 1 " = $\frac{12}{11}$ "

" " 25 " = $\frac{12 \times 25}{11}$ "

= $27\frac{3}{11}$ "

past 3 o'clock.

51. What is the first time after 6 that the hands of a clock are 8 minute-divisions apart? the second time?

52. It is between four and five, and the hands of a watch are equally distant from the figure 4; what is the first time? the second time?

53. At what time from 4 to 6 are the hands of a watch exactly opposite each other?

54. What is the first time after 1, 3, 6 and 8 respectively that the hands of a clock are equally distant from these figures?

55. At what time between 4 and 5 o'clock are the hands of a watch at right angles to each other?

56. At what time between 1 and 2 o'clock are the hands of a watch opposite to each other?

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57. The sum of two numbers is 368, and their difference is 146; what are the numbers?

Greater number + less number = 368;
Greater number − less number = 146;
∴ 2 × greater number . . = 514;
∴ greater number . . . = 257;
Also 2 × less number . . . = 222;
∴ less number . . . . . = 111.

The rate of rowing down stream is the sum of the rate in still water and the rate of the stream; the rate of rowing up stream is the rate in still water diminished by the rate of the stream.

58. *A* can row 10 mi. down stream in 4 hours, but he requires 5 hours to row back again; find the rate of the stream.

Rate in still water + rate of stream = $2\frac{1}{2}$ mi. per hr.
Rate in still water − rate of stream = 2 "
∴ 2 × rate of stream = $\frac{1}{2}$ "
∴ rate of stream = $\frac{1}{4}$ "

59. A steamboat sails 60 miles down stream in $7\frac{1}{2}$ hours, but requires 8 hours to sail back. Find the rate of the boat in still water.

60. *A* can row $4\frac{1}{2}$ miles down stream in an hour. Without the aid of the stream it would have taken him an hour and a half. How long would it take him to return?

61. A crew can row up a stream a certain distance in 64 min. and back again in 60 min.; find the distance, the rate of the stream being half a mile an hour.

Distance down the stream in 1 hr. = $\frac{15}{16}$ of dist. up in 1 hr.
Therefore $\frac{1}{16}$ of distance = 2 rate of stream.
= 1 mile;
therefore distance = 16 miles.

62. A crew can row down a stream a certain distance in 60 minutes, and up the same distance in 64 minutes; find the rate of the stream in terms of the distance.

63. A man can row down a stream a certain distance and back again in 3 hours, and the rate of the stream is half his rate; how long is he going down and how long going up?

64. A man can row 8 miles down a stream in one hour and twenty minutes, and back again in two hours; find the rate per hour and the rate of the stream.

65. A man receives $3 a day for his labor, and pays $1 a day for his board. At the expiration of 50 days he has saved $70. How many days was he idle?

Amt. he would have rec'd had he worked full time = $100;
Amount actually received . . . . . = $ 70;
Lost through idleness . . . . . = $ 30;
Days idle at $3 per day . . . . . = 10 dys.

66. A boy agreed to carry 200 oranges to market at 1½ cts. each, on condition that he should forfeit 4 cts. for each one he eat; he received $2.60; how many did he eat?

67. A girl agreed to carry 80 glasses to the depot for 2½ cts. each, on condition that she should forfeit 12 cts. for every one she broke; she received 20 cts.; how many did she carry safely?

68. 12 men hire a boat, but taking in 8 more the expenses of each is diminished by 50 cts.; what did the boat cost?

Diminution of expenses of 12 men at 50 cts. each = $6;
This is paid by the 8 men, *i.e.*, each paying . = 75 cts.
Then 20 men at 75 cts. each . . . . . = $15.

69. Sixteen men hire a coach for a certain sum, but by taking in 4 more persons the expenses of each is diminished by 12½ cents; what did they pay for the coach?

70. A company of 12 persons agree to buy a manufactory, but 4 of the company withdrawing, the investment of each is increased by $180; what did the manufactory cost?

71. How far may a person ride in a coach going at the rate of 12 miles an hour, provided he is gone but 10 hours, and walks back at the rate of 8 miles an hour?

Distance travelled by coach . . in $\frac{1}{12}$ hour = 1 mile.
" returning . . in $\frac{1}{5}$ " = 1 "
" going and returning in $\frac{5}{24}$ " = 1 "
" " in 1 " = $\frac{24}{5}$ "
" " in 10 " = $\frac{10 \times 24}{5}$
= 48 mls.

72. How far may a person ride in the cars, going at the rate of 24 miles an hour, supposing he is gone 14 hours, and returns at the rate of 4 miles an hour?

73. How far may a person ride in a carriage going at the rate of 8 miles an hour, provided he is gone 11 hours and walks back at the rate of 3 miles an hour?

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74. A hare is 80 leaps before a hound, and takes 4 leaps while the hound takes 2, but two of the hound's leaps equal 5 of the hare's; how many leaps will the hound take to catch the hare?

5 of hare's leaps = 2 of hound's;
but the hare takes 4 leaps while the hound takes 2 leaps;
∴ on every 2 leaps the hound gains 1 leap.
Hence to gain 80 leaps of hare the hound must take 160 leaps.

75. A hare is 120 leaps before a hound, and takes 5 leaps while the hound takes 3, but three of the hound's leaps equal 7 of the hare's; how many leaps will the hound take to catch the hare?

76. A boy is 95 steps before a man, and takes 3 steps to the man's 2, but 3 of the man's steps equal 6 of the boy's; how many steps will each take before being together?

ANSWERS.

Exercise I.—Page 2.

1. 5; 5; 9. 2. 1; 1 book; 1 ball.
3. 6, 7, 3, 4, 2 are abstract; 8 books, 9 men, 5 apples, 1 cent are concrete. 4. 1 mile; 1 mile; 1; 1 cent.
5. 3 apples, 7 apples, and 6 apples; 4 boys and 9 boys; 7, 9 and 8; 2 cents and 5 cents; 4 girls and 5 girls.

Exercise II.—Page 3.

1. 7; 9; 4; 2. 2. 36; 84; 20; 69.
3. 44; 70; 96; 16. 4. 14; 12; 39; 56. 5. 48; 97; 36; 60.
6. Seven; eleven; fifteen; nineteen; fifty-nine; eighty-four; ninety-six; ninety-eight.
7. Seventy-one; twelve; twenty-eight; ninety-one; forty-four; seventeen; twenty-two; thirty-four.
8. Twenty; thirty-seven; forty-eight; seventy-six; ninety-nine; sixty-nine; seventy; eighty-seven.
9. Fourteen; thirty-five; eighty-nine; seventy-eight; fifty-four; forty-nine; fifty; thirteen.
10. Ninety; eighty; thirty-nine; twenty-eight; eleven; nineteen; twenty-seven; thirty-one.

Exercise III.—Page 4.

1. 149; 308; 974. 2. 200; 420; 694.
3. 560; 908; 444. 4. 735; 960; 406.
5. 309; 687; 472.
6. Two hundred and seven; three hundred and seventy-one; one hundred and eighty-five; one hundred and ninety; three hundred and sixty-eight.
7. Five hundred and seventy; four hundred and seventy-two; eight hundred and seven; nine hundred and nine; nine hundred and ninety.
8. Three hundred and sixty-eight; five hundred and eighty-four; seven hundred and sixty; three hundred and twenty-one; nine hundred and ninety-nine.

9. Three hundred and ninety-four; seven hundred and eighty-six; four hundred and seventy-five; seven hundred and eighty-two; seven hundred.

10. Five hundred and six; three hundred; four hundred and seven; seven hundred and forty; three hundred and ninety-seven.

Exercise IV.—Page 5.

1. 6006; 4300; 9080.
2. 3700; 7906; 3084.
3. 64009; 807068; 700316.
4. 4030097; 809007039; 586000007.
5. 8000000000; 64007000024; 4004000004.
6. 408003009; 74000074004; 500000000500.
7. 80070000000; 800000008; 300000300090.
8. 57700000080; 11000011; 19000014000.
9. 7000000000070; 400000001; 600600000000600.
10. 99000000000008; 700070007000; 16016000000016.

11. Seven thousand and seventy-seven; eighty-five thousand and seventy-nine; fifty-six thousand, nine hundred and fifty; four hundred and seventy-three thousand, six hundred and twenty-eight.

12. Fifty-six thousand, four hundred and eighteen; seven hundred and eighty-four thousand and six; four hundred thousand, five hundred and seven; three hundred and sixty thousand and four.

13. Three hundred thousand and seventy-one; nine hundred and one thousand and seven; seven hundred and twenty thousand and nine; one hundred and eighty-two thousand and ten.

14. Three millions, one hundred and forty thousand and six; fifty millions and six hundred; three billions, six hundred millions, ten thousand and seventy.

15. Fifty-one billions, six hundred and thirty-six millions, two hundred and seven thousand, six hundred and forty; seventy billions and one hundred; nine hundred and twenty billions, seventy millions, seventy thousand and seventy.

Exercise V.—Page 7.

1. XIX; XXIV; XLIX; LXXXIV; XCIX.
2. CLXXXVII; CCVIII; DCCLXXXI; CMLXII; CMXCIX.
3. MCCCI; MCCCXC; MDCLXXXIV; MDCCCXV; MDCCCLXXVIII.
4. 44; 69; 94; 71.
5. 99; 129; 177.
6. 555; 1604; 1819; 1090.

Exercise VI.—Page 11.

1. 47 horses.	2. 98 boys.	3. 39 girls.	4. 978.
5. 956.	6. 898.	7. 889.	8. 879.
9. 979.	10. 697.	11. 798.	12. 998.
13. 898.	14. 879.	15. 889.	16. 8589
17. 9879.	18. 8989.	19. 9989.	20. 98878.
21. 87988.	22. 88998.	23. 79988.	24. 797898.
25. 768989.	26. 789689.	27. 988989.	

Exercise VII.—Page 12.

1. 79 cents.	2. 88 trees.	3. 968 acres.
4. 796 dollars.	5. 989 miles.	6. 969 yards.
7. 878 bales.	8. 8989 dollars.	9. 97989 dollars.
10. 899898 persons.		

Exercise VIII.—Page 14.

1. 113 dollars.	2. 78 cents.	3. 152 boys.	4. 145 girls.
5. 146.	6. 247.	7. 162.	8. 161.
9. 217.	10. 213.	11. 1861.	12. 1357.
13. 1915.	14. 1954.	15. 1931.	16. 1759.
17. 2704.	18. 1656.	19. 1951.	20. 1976.
21. 1842.	22. 2141.	23. 23878.	24. 18294.
25. 16954.	26. 14978.	27. 15113.	28. 16046.
29. 24459.	30. 31405.	31. 29377.	32. 21232.
33. 22825.	34. 29165.	35. 256648.	36. 226871.
37. 166581.	38. 338306.	39. 2033781.	40. 199859286.
41. 24692.	42. 25879.	43. 27265.	44. 24447.
45. 23378.	46. 238390.	47. 246818.	48. 81148.
49. 75953.	50. 103618.	51. 41081.	

Exercise IX.—Page 15.

1. 222 dollars.	2. 1660 acres.	3. 120 days.
4. 1061 miles.	5. 936 pounds.	6. 7428 bushels.
7. 3111 acres.	8. 633 dollars.	9. 2104 pages.
10. 2237 dollars.	11. 1173 dollars.	
12. B, 601 dollars; C, 1066 dollars; 2132 dollars.		
13. 31080 dollars.		

Exercise X.—Page 19.

1. 313.	2. 241.	3. 251.	4. 402.
5. 644.	6. 464.	7. 143.	8. 305.
9. 344.	10. 304.	11. 733.	12. 530.

13. 4442.	14. 5022.	15. 2223.	16. 2001.
17. 2530.	18. 4422.	19. 5512.	20. 2734.
21. 5024.	22. 6257.	23. 1361.	24. 4623.
25. 423.	26. 60224.	27. 36275.	28. 31216.
29. 5082.	30. 43264.	31. 36425.	32. 35137.
33. 66243.	34. 75331.	35. 61161.	36. 40663.
37. 262265.	38. 64153.	39. 35422.	40. 77443.
41. 161116.	42. 741551.	43. 21353.	44. 44516.
45. 57234.	46. 364.	47. 233.	48. 228.
49. 322.	50. 432.	51. 2533.	52. 1243.
53. 6216.	54. 83136.	55. 56454.	

Exercise XI.—Page 20.

1. 43 girls.	2. 44 cents.	3. 16 dollars.
4. 34 runs.	5. 44 questions.	6. 43 dollars.
7. 33 dollars.	8. 2112 dollars.	9. 14442 dollars.

Exercise XII.—Page 23.

1. 325.	2. 373.	3. 262.	4. 293.
5. 255.	6. 144.	7. 184.	8. 256.
9. 263.	10. 286.	11. 362.	12. 309.
13. 168.	14. 365.	15. 266.	16. 169.
17. 69.	18. 68.	19. 458.	20. 178.
21. 298.	22. 175.	23. 197.	24. 118.
25. 339.	26. 468.	27. 177.	28. 497.
29. 479.	30. 293.	31. 1497.	32. 2858.
33. 4188.	34. 948.	35. 1933.	36. 2919.
37. 4944.	38. 2857.	39. 5339.	40. 1299.
41. 1359.	42. 5247.	43. 2279.	44. 5263.
45. 3784.	46. 5682.	47. 4469.	48. 1789.
49. 11844.	50. 19528.	51. 52888.	52. 35498.
53. 49289.	54. 25012.		

Exercise XIII.—Page 24.

1. 8 dollars. 2. 77 yards. 3. 560 dollars. 4. 1803.
5. 3251. 6. 344 cents. 7. 37 quarts. 8. 175 dollars.
9. 506 doliars. 10. 375 acres; 12021 dollars.

Exercise XIV.—Page 24.

1. 177.	2. 739.	3. 1811.	4. 691.
5. 2262.	6. 520.	7. 2152.	8. 22326.
9. 77865.	10. 3598.		

Exercise XV.—Page 25.

1. 357 dollars. 2. Lost 632 dollars. 3. 853288
4. 10534. 5. 171 dollars. 6. 956.
7. 814 feet. 8. John 28, James 32. 9. 41265.
10. 6628.

Exercise XVI.—Page 29.

1. 14864.	2. 16864.	3. 216936.	4. 368492.
5. 195 boys.	6. 282 cents.	7. 959 cows.	8. 1488 apples.
9. 2106 girls.	10. 1890.	11. 3360.	12. 3070.
13. 23526.	14. 47901.	15. 43710.	16. 78112.
17. 53838.	18. 70340.	19. 72028.	20. 661672.
21. 153132.	22. 630855.	23. 352794.	24. 646857.
25. 53936.	26. 54360.	27. 432481.	28. 364704.
29. 428215.	30. 1023024.	31. 3417355.	32. 6044346.
33. 6283784.	34. 2217177.	35. 7865190.	36. 9162527.
37. 10860916.	38. 9091656.	39. 10743888.	

Exercise XVII.—Page 30.

1. 16280 cents. 2. 185430 cents. 3. 2709 dollars.
4. 15215 dollars. 5. 336 sheep; 392 dollars.
6. 94 pounds; 2610 cents; 752 cents; 658 cents.
7. 3402 dollars; 224 dollars; 3626 dollars; 2237 dollars.
8. 1320 paragraphs; 11880 lines; 95040 words; 475200 letters.
9. 296 cents; 414 cents; 710 cents; 118 cents.
10. 1096 cents; 2211 cents; 3310 cents; 1118 cents.

Exercise XVIII.—Page 31.

1. 11950.	2. 40992.	3. 118377.
4. 66738.	5. 482544.	6. 340488.
7. 189945.	8. 230896.	9. 134010.
10. 62550.	11. 183576.	12. 1348560.
13. 98560 yards.	14. 68520 feet.	15. 8532 cents.
16. 459 days.	17. 3000 dollars.	18. 8505 dollars.
19. 19845 dollars.	20. 40320 min.	

Exercise XIX.—Page 33.

1. 472440.	2. 300720.	3. 236196.
4. 562650.	5. 724885.	6. 6608822.
7. 6586169.	8. 6509916.	9. 1194872.

10. 4127874. 11. 9781440. 12. 11961586.
13. 14821755. 14. 25581580. 15. 23120856.
16. 81362385. 17. 29455710. 18. 31259060.
19. 70132632. 20. 14069499. 21. 41316048.
22. 26514000. 23. 42741832. 24. 16765686.
25. 66093951. 26. 217702278. 27. 163588743.
28. 307551216. 29. 276010344. 30. 114297351.
31. 348112465. 32. 283036032. 33. 671374392.
34. 2139927997. 35. 1627916724. 36. 1244124564.
37. 341614192. 38. 3481804952. 39. 4141417504.
40. 903556918.

Exercise XX.—Page 34.

1. 127405. 2. 6317608. 3. 1960452. 4. 6825456.
5. 63366216. 6. 6749472. 7. 8214206. 8. 25996104.
9. 80071992. 10. 738110274.

Exercise XXI.—Page 34.

1. 283800. 2. 592900. 3. 60744600. 4. 11887500.
5. 243000. 6. 258000. 7. 11214000. 8. 4096000.
9. 422500. 10. 627000000. 11. 64610000. 12. 488000000.

Exercise XXII.—Page 34.

1. 454560 sheets. 2. 195559 yards. 3. 6125 dollars.
4. 1653 yards. 5. 3915648 pounds. 6. 1228275 dollars.
7. 263952 apples. 8. 915750 pages. 9. 61275 yards.
10. 193662 yards. 11. 44100 dollars. 12. 12180 dollars.
13. 1246420 dollars. 14. 3926000 dollars. 15. 262800 barrels.
16. 7080320 dollars. 17. 277107850 dollars 18. 372480 hills.
19. 89784 yards. 20. 7344 miles.

Exercise XXIII.—Page 36.

1. 608 dollars. 2. 162764 men. 3. 530229 gal.
4. 2773 dollars. 5. 12652 cents. 6. 7455 dollars; 9940 dollars. 7. The horses; 2532 dollars.
8. *A*'s 706800; *B*'s 1126125 letters. 9. 18750 cents.
10. 116550 cents. 11. 457047. 12. 944 days.
13. 1441 dollars. 14. House, 2800 dollars; Farm, 2975 dollars. 15. Loss 254 dollars.
16. Gain 3100 dollars. 17. 17582 dollars. 18. 9000 dollars.
19. 8110 dollars. 20. 10690 dollars. 21. 589 dollars.

Exercise XXIV.—Page 42.

1. 18.	2. 29.	3. 27.	4. 46.	5. 48.	6. 192.
7. 155.	8. 241.	9. 291.	10. 325.	11. 213.	12. 191.
13. 233.	14. 144.	15. 187.	16. 147.	17. 170.	18. 195.
19. 149.	20. 167.	21. 122.	22. 141.	23. 154.	24. 162.
25. 139.	26. 112.	27. 114.	28. 119.	29. 138.	30. 137.
31. 112.	32. 117.	33. 118.	34. 122.	35. 124.	36. 52.
37. 64,	38. 96.	39. 82.	40. 74.		

Exercise XXV.—Page 43.

1. 228.	2. 368.	3. 274.	4. 187.	5. 269.
6. 245.	7. 272.	8. 174.	9. 138.	10. 246.
11. 223.	12. 171.	13. 182.	14. 255.	15. 275.
16. 128.	17. 156.	18. 183.	19. 144.	20. 206.
21. 184.	22. 204.	23. 243.	24. 152.	25. 109.
26. 123.	27. 147.	28. 129.	29. 157.	30. 168.
31. 163.	32. 187.	33. 156.	34. 153.	35. 176.
36. 139.	37. 108.	38. 109.	39. 129.	40. 144.
41. 246.	42. 556.	43. 419.	44. 609.	45. 1223.
46. 367.	47. 676.	48. 1208.	49. 1337.	50. 1410.
51. 907.	52. 457.	53. 947.	54. 3669.	55. 13879.
56. 988.	57. 442.	58. 285.	59. 7032.	60. 7484.

Exercise XXVI.—Page 44.

1. 59 oranges.	2. 173 days' work.	3. 918 pounds.
4. 231 yards.	5. 96 rods.	6. 91 cents.
7. 15 bushels.	8. 123.	9. 6052 bushels.
10. 8 cords.		

Exercise XXVII.—Page 45.

1. $2177\frac{1}{3}$.	2. $1248\frac{4}{5}$.	3. 9606.	4. $1640\frac{5}{7}$.
5. $12317\frac{1}{8}$.	6. $69049\frac{1}{6}$.	7. $420061\frac{1}{4}$.	8. $572004\frac{3}{8}$.
9. $777070\frac{3}{4}$.	10. $729581\frac{3}{10}$.	11. 1398260.	12. 7400061.
13. 823956.	14. $6273804\frac{1}{3}$.	15. $1238753\frac{6}{7}$.	16. $587296\frac{1}{6}$.
17. 20073842.	18. 37037048.	19. $847085\frac{3}{4}\frac{1}{4}$.	20. 7298426.
21. 7480093.	22. $2730956\frac{1}{2}\frac{9}{0}$.	25. $8841326\frac{1}{4}\frac{1}{4}$.	24. 92506025.
25. $4560387\frac{5}{10}$.	26. 37376008.		

Exercise XXVIII.—Page 46.

1. 432 barrels.	2. 1256 pounds.	3. 6226 dollars.
4. $11\frac{1}{8}$ dollars.	5. 8 dollars.	6. 2423 minutes.
7. $11\frac{1}{4}$ apples.	8. $130\frac{1}{4}$ pounds.	9. $125\frac{1}{2}$ dollars.
10. $52\frac{1}{4}$ weeks.	11. 252.	12. $689\frac{3}{4}$ acres.
13. $15745\frac{1}{8}$ pounds.	14. 548 bricks.	

Exercise XXIX.—Page 48.

1. $24\frac{20}{31}$.
2. $8\frac{39}{41}$.
3. $18\frac{33}{53}$.
4. $74\frac{53}{61}$.
5. $39\frac{50}{74}$.
6. $588\frac{51}{65}$.
7. $531\frac{45}{56}$.
8. $945\frac{27}{76}$.
9. $554\frac{66}{122}$.
10. $49\frac{49}{801}$.
11. $231\frac{231}{346}$.
12. $498\frac{153}{336}$.
13. $375\frac{375}{425}$.
14. $2259\frac{1}{239}$.
15. $5050\frac{99}{198}$.
16. 2831.
17. $6205\frac{162}{1449}$.
18. $6200\frac{160}{1560}$.
19. 3746.
20. $2025\frac{372}{1736}$.
21. $4998\frac{432}{1976}$.
22. $9710\frac{140}{2492}$.
23. 4321.
24. 4671.
25. 3180.
26. 3615.
27. 1142.
28. $7277\frac{94677}{786017}$.
29. 2567.
30. 50444.

Exercise XXX.—Page 49.

1. 43 days.
2. 38 days.
3. 1090 feet.
4. 32 dollars.
5. 129 years.
6. 123 dollars.
7. 46 dollars.
8. 545 bales.
9. $343\frac{5}{17}$ miles.
10. 2075 barrels.

Exercise XXXI.—Page 50.

1. $1734\frac{9}{27}$.
2. $1366\frac{44}{64}$.
3. $1549\frac{61}{63}$.
4. $307\frac{38}{45}$.
5. $246925\frac{21}{35}$.
6. $149147\frac{13}{49}$.
7. $349\frac{17}{25}$.
8. $696\frac{26}{96}$.
9. $1082\frac{6}{81}$.
10. $4236\frac{57}{99}$.
11. 2570.
12. $5599\frac{89}{144}$.

Exercise XXXII.—Page 51.

1. $24\frac{5}{30}$.
2. $127\frac{22}{69}$.
3. $12\frac{242}{700}$.
4. $326\frac{61}{300}$.
5. $804\frac{9}{90}$.
6. $1183\frac{98}{800}$.
7. $2\frac{3886}{17000}$.
8. $5\frac{32}{3600}$.
9. $30\frac{97541}{123000}$.
10. $153\frac{55}{6400}\frac{51}{}$.
11. $673\frac{1773}{2500}$.
12. $632\frac{62379}{150000}$.

Exercise XXXIII.—Page 51.

1. 108 yards.
2. 65 hours.
3. 123 pounds.
4. 30 pounds.
5. 42 bushels.
6. 235 dollars.
7. 1378 quarters.
8. 237 bushels.
9. 43 bushels.
10. 38 miles.

Exercise XXXIV.—Page 52.

1. 8814.
2. 129.
3. 233289.
4. 348.
5. 186.
6. 272.
7. 10005100.
8. 19052.
9. 194 and 86.
10. 784623.

Exercise XXXV.—Page 53.

1. 367 acres.
2. 2310 dollars.
3. 845 dollars.
4. 15 weeks.
5. 44 dollars.
6. $65\frac{23}{28}$ cents.
7. 1650 barrels.
8. 24 months.
9. 551 dollars.
10. 1060 dollars.
11. 41600 c. ft.
12. 217 sheep.
13. 520 dollars.
14. 30 hours.
15. 240 cents.

16. 54 cents. 17. 56 dollars. 18. 96 dollars.
19. 100 dollars. 20. 40 cents. 21. 12 dollars.
22. 6880 dollars. 23. 23725 days. 24. 7056 pounds.
25. 5640 yards. 26. 365 acres. 27. 31250 dollars.
28. 954 dollars. 29. 1971 bushels. 30. 15 days.
31. 12 days. 32. 32 days. 33. 361 days.
34. 50 days. 35. 90 days. 36. 48 days.
37. 6 days. 38. 10 days. 39. 119 days.
40. 28 men. 41. 105 men. 42. 21 men.
43. 108 men. 44. 25 men. 45. 114 men.
46. 56 men. 47. 72 men. 48. 150 men.
49. 15 beggars.

Examination Papers.—Page 57.

I.

2. 488979. 3. 944813. 4. 7706307420. 5. 1116 dollars.

II.

2. 29900000. Twenty-nine millions, nine hundred thousand.
3. 846055. 4. 2699; 329; DCCCLXXXVIICMLXXI.
5. 16 bushels.

III.

3. 51365636568. 5. 580 acres; 61 dollars.

IV.

4. 228 dollars. 5. 608 sheep.

V.

3. 87. 4. 86. 5. 53 dollars.

VI.

2. 3571 dollars. 3. 112. 4. 180 acres; 36 dollars.
5. 24 days.

VII.

2. 40831 dollars. 3. 663 miles. 4. 247. 5. 6525 dollars.

VIII.

1. 4700 dollars. 2. 973. 3. 31 dollars.
4. 201 cents. 5. 964 miles; 1181 miles.

Exercise XXXVIII.—Page 64.

1. $1163.55. 2. $1864.07. 3. $3220.65. 4. $1624.90.
5. $99.05. 6. $82.28. 7. $5232.44. 8. $22.

Exercise XXXIX.—Page 66.

1. $94.58.	2. $58.75.	3. $43.19.	4. $592.61.
5. $5170.64.	6. $23.79.	7. $261.07.	8. $7915.80.
9. $4.08.	10. $50.37.	11. $1790.63.	12. $49.15.

Exercise XL.—Page 67.

1. $391.85.	2. $1482.96.	3. $926.25.	4. $97670.
5. $6562.50.	6. $3522.75.	7. $157.50.	8. $27.75.
9. $3364.20.	10. $16.80.	11. $247.	12. $169.
13. $1794.	14. $722.16.	15. $360.	16. $3.51.
17. $261.25.	18. $425.25.		

Exercise XLI.—Page 69.

1. $12.72.	2. $21.37.	3. $18.17.	4. $26.34.
5. $7.89.	6. 60.50\frac{1}{35}$.	7. $10.40.	8. 73 sheep.
9. $2.22.	10. 6.	11. 365 days.	12. 16 pieces.

Exercise XLII.—Page 71.

1. $32.20.	2. $11.96.	3. $4.35.	4. $27.76.	5. $47.02
6. $889.77.	7. $14.24.	8. $771.51.	9. $18.78.	10. $3.31.

Examination Papers.—Page 72.

I.

2. 70 cents.	3. $5.15.	4. 100.	5. 15 times more.

II.

2. $122.	3. $281.52.	4. 50 tons.	5. $1.10.

III.

2. 45 yards.	3. 85 votes.	4. $1191.75.	5. 1760.

IV.

1. 476 yards; 30 cents.	2. 400 bushels.	3. 1560 pair.
4. 100 days.	5. 1100; 430.	

Exercise XLIII.—Page 76.

1. 2, 2, 2, 2, 3.	2. 2, 2, 2, 3, 3.	3. 3, 3, 3, 3.
4. 2, 2, 3, 3, 3.	5. 5, 5, 7.	6. 2, 3, 3, 3, 5.
7. 2, 2, 2, 2, 2, 5.	8. 5, 5, 13.	9. 3, 11, 13.
10. 2, 2, 3, 23.		11. 2, 2, 2, 2, 2, 5, 5.
12. 2, 2, 3, 3, 5.	13. No prime factor.	14. 2, 2, 2, 2, 3, 7.
15. 3, 3, 5, 19.	16. 3, 5, 7, 11.	17. 2 and 5.
18. 3.	19. 7 and 3.	20. 2, 2, 2, 3 and 5.

Exercise XLIV.—Page 77.

1. 2.	2. 2.	3. 12.	4. 18.
5. 30.	6. 72.	7. 20 yards.	8. 72 bushels.
9. $22.	10. 75 yards.	11. $2.	12. 1440.

Exercise XLV.—Page 78.

1. 5.	2. 4.	3. 8.	4. 14.	5. 10.
6. 42.	7. 24.	8. 11.	9. 75.	10. 144.

11. 8 feet. 12. 21 feet. 13. 16 feet. 14. 8 quarts. 15. 45 pears
16. 3, 11, or 33 pupils in each section.

Exercise XLVI.—Page 80.

1. 23.	2. 37.	3. 41.	4. 56.
5. 45.	6. 61.	7. 42.	8. 11.
9. 813.	10. 630.	11. Prime.	12. 21.

13. 1841b. 14. 7 and 12. 15. 1 ac. or 2 ac. or 3 ac. or 6 ac.
16. 60 ft.; *A*, 10; *B*, 12; *C*, 15. 17. 15 feet. 635 rails. 127 posts.

Exercise XLVII.—Page 83.

1. 30.	2. 60.	3. 36.	4. 150.
5. 360.	6. 180.	7. 360.	8. 770.
9. 2520.	10. 1512.	11. 1680.	12. 16800.
13. 1800.	14. 720720.	15. 50702925.	16. 173.
17. $2100.	18. 360 bu.	19. 240 cents.	20. 84 bushels.

21. 120 days. 22. 301 marbles.

Examination Papers.—Page 84.

I.

1. 611, 707, and 1089 are comp.; 643, 757 and 991 are prime.
2. 8. 3. $3048. 4. 643. 5. 25 acres.

II.

2. 25. 3. 46. 4. 1680 marbles. 5. 47100 holes.

III.

1. 15, 16, 17, and 18. 2. 900 acres. 3. 9 cents.
4. 356. 5. 9672 rails.

IV.

1. 75 cents. 2. $1080. 3. 3600. 4. 10565999.
5. 1267994828100.

V.

1. 10296. 2. 240. 3. 257. 4. 5 and 4.

Exercise XLVIII.—Page 90.

1. $\frac{25}{7}$. 2. $\frac{21}{5}$. 3. $\frac{65}{7}$. 4. $\frac{41}{9}$.
5. $\frac{133}{12}$. 6. $\frac{99}{8}$. 7. $\frac{431}{12}$. 8. $\frac{1409}{17}$.
9. $\frac{1190}{23}$. 10. $\frac{1639}{19}$. 11. $\frac{1090}{11}$. 12. $\frac{7741}{99}$.
13. $\frac{360237}{152}$. 14. $\frac{718129}{2306}$. 15. $\frac{232477}{178}$. 16. $\frac{24326084}{973}$.
17. $\frac{10111112}{101}$. 18. $\frac{28904819}{996}$.

Exercise XLIX.—Page 91.

1. $3\frac{4}{5}$. 2. $5\frac{1}{6}$. 3. $6\frac{3}{7}$. 4. $7\frac{5}{10}$.
5. 33. 6. 17. 7. $12\frac{*}{16}$. 8. $65\frac{7}{15}$.
9. $16\frac{5}{45}$. 10. 13. 11. 28. 12. $51\frac{13}{19}$.
13. $32\frac{55}{136}$. 14. $100\frac{19}{59}$. 15. $515\frac{12}{33}$. 16. $676\frac{12}{48}$.
17. $36\frac{666}{694}$. 18. $522\frac{90}{139}$.

Exercise L.—Page 91.

1. $\frac{3}{4}$. 2. $\frac{2}{3}$. 3. $\frac{2}{5}$. 4. $\frac{2}{3}$. 5. $\frac{5}{6}$. 6. $\frac{5}{7}$.
7. $\frac{5}{7}$. 8. $\frac{9}{10}$. 9. $\frac{1}{3}$. 10. $\frac{11}{12}$. 11. $\frac{15}{77}$. 12. $\frac{7}{9}$.
13. $\frac{61}{71}$. 14. $\frac{3}{4}$. 15. $\frac{1623}{3223}$. 16. $\frac{3167}{4177}$. 17. $\frac{19}{67}$. 18. $\frac{4}{7}$.
19. $\frac{7}{11}$. 20. $\frac{4}{15}$. 21. $\frac{11}{19}$. 22. $\frac{2}{3}$. 23. $\frac{19}{23}$. 24. $\frac{241897}{25}$.

Exercise LI.—Page 93.

1. $\frac{20}{77}$. 2. $\frac{3}{11}$. 3. $3\frac{3}{16}$. 4. $1\frac{19}{36}$. 5. $\frac{8}{21}$. 6. $\frac{5}{66}$.
7. $\frac{10}{21}$. 8. $\frac{5}{9}$. 9. $\frac{9}{50}$. 10. $\frac{10}{33}$. 11. $\frac{1}{9}$. 12. $\frac{3}{32}$.
13. $\frac{51}{55}$. 14. 3. 15. $2\frac{19}{28}$. 16. $\frac{17}{72}$. 17. $1\frac{1}{4}$. 18. $1\frac{2}{5}$.

Exercise LII.—Page 93.

1. $\frac{1}{2}$. 2. $\frac{5}{8}$. 3. $\frac{9}{25}$. 4. $\frac{2}{15}$. 6. $\frac{1}{8}$. 7. $12\frac{1}{2}$ acres. 8. \$8750.

Exercise LIII.—Page 94.

1. $\frac{27}{45}, \frac{35}{45}$ 2. $\frac{60}{70}, \frac{63}{70}$. 3. $\frac{88}{99}, \frac{63}{99}$.
4. $\frac{70}{105}, \frac{63}{105}, \frac{75}{105}$. 5. $\frac{75}{105}, \frac{70}{105}, \frac{84}{105}$. 6. $\frac{36}{60}, \frac{40}{60}, \frac{45}{60}$.
7. $\frac{252}{420}, \frac{140}{420}, \frac{120}{420}, \frac{105}{420}$. 8. $\frac{4680}{6552}, \frac{2457}{6552}, \frac{2912}{6552}, \frac{5544}{6552}$.
9. $\frac{2376}{3960}, \frac{2475}{3960}, \frac{1760}{3960}, \frac{2880}{3960}$.

Exercise LIV.—Page 95.

1. $\frac{20}{30}, \frac{25}{30}, \frac{18}{30}$. 2. $\frac{4}{8}, \frac{6}{8}, \frac{7}{8}$. 3. $\frac{35}{40}, \frac{36}{40}, \frac{14}{40}$.
4. $\frac{20}{36}, \frac{21}{36}, \frac{22}{36}$. 5. $\frac{30}{36}, \frac{16}{36}, \frac{21}{36}$. 6. $\frac{45}{60}, \frac{42}{60}, \frac{50}{60}$.
7. $\frac{15}{60}, \frac{35}{60}, \frac{33}{60}$. 8. $\frac{25}{30}, \frac{24}{30}, \frac{20}{30}$. 9. $\frac{35}{40}, \frac{30}{40}, \frac{32}{40}$.
10. $\frac{20}{10}, \frac{3}{10}, \frac{8}{10}$. 11. $\frac{15}{6}, \frac{20}{6}, \frac{5}{6}$. 12. $\frac{7}{9}, \frac{32}{9}, \frac{45}{9}$.
13. $\frac{48}{16}, \frac{72}{16}, \frac{1}{16}$. 14. $\frac{147}{168}, \frac{60}{168}, \frac{70}{168}, \frac{52}{168}$.
15. $\frac{44}{77}, \frac{35}{77}, \frac{9}{77}, \frac{462}{77}$. 16. $\frac{28}{105}, \frac{455}{105}, \frac{60}{105}, \frac{210}{105}$.
17. $\frac{336}{576}, \frac{408}{576}, \frac{176}{576}, \frac{405}{576}$. 18. $\frac{405}{180}, \frac{1296}{180}, \frac{580}{180}, \frac{825}{180}$.

Exercise LV.—Page 96.

1. $\frac{4}{7}$. 2. $\frac{25}{27}$. 3. $\frac{10}{21}$. 4. $\frac{11}{18}$.
5. $\frac{17}{20}$. 6. $\frac{17}{21}$. 7. $\frac{5}{34}$. 8. $\frac{7}{40}$.
9. $\frac{13}{15}$. 10. $\frac{10}{20}$; $\frac{16}{35}$. 11. $\frac{16}{33}$; $\frac{9}{22}$. 12. $\frac{2}{6}$; $\frac{3}{16}$.
13. $\frac{9}{10}$; $\frac{21}{25}$. 14. $\frac{13}{15}$; $\frac{17}{21}$. 15. $\frac{8}{9}$; $\frac{5}{6}$.
16. $\frac{4}{84}$, $\frac{3}{18}$, $\frac{5}{27}$, $\frac{17}{81}$, $\frac{2}{3}$. 17. $\frac{1}{2}$, $\frac{13}{20}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$.

Exercise LVI.—Page 98.

1. $1\frac{5}{12}$. 2. $1\frac{27}{40}$. 3. $\frac{29}{36}$. 4. $1\frac{7}{12}$. 5. $1\frac{39}{40}$. 6. $\frac{107}{200}$.
7. $2\frac{2}{7}$. 8. $2\frac{7}{16}$. 9. $2\frac{9}{35}$. 10. $1\frac{91}{180}$. 11. $3\frac{341}{420}$. 12. $\frac{37}{51}$.

Exercise LVII.—Page 98.

1. $10\frac{11}{30}$. 2. $10\frac{7}{16}$. 3. $10\frac{29}{42}$. 4. $5\frac{23}{60}$. 5. $4\frac{109}{666}$. 6. $46\frac{8}{15}$.
7. $22\frac{89}{180}$. 8. $29\frac{7}{18}$. 9. $12\frac{5}{8}$. 10. $21\frac{17}{20}$. 11. $12\frac{5}{9}$. 12. $16\frac{3}{9}$.

Exercise LVIII.—Page 100.

1. $\frac{4}{35}$. 2. $\frac{17}{45}$. 3. $\frac{13}{36}$. 4. $\frac{1}{20}$. 5. $\frac{1}{16}$. 6. $\frac{2}{105}$.
7. $\frac{7}{60}$. 8. $\frac{409}{700}$. 9. $\frac{77}{171}$. 10. $\frac{7}{18}$. 11. $\frac{1}{16}$. 12. $\frac{1}{42}$.

Exercise LIX.—Page 100.

1. $1\frac{23}{35}$. 2. $1\frac{1}{90}$. 3. $3\frac{7}{12}$. 4. $2\frac{63}{83}$. 5. $1\frac{7}{12}$. 6. $1\frac{1}{4}$.
7. $2\frac{1}{42}$. 8. $2\frac{5}{56}$. 9. $4\frac{1}{7}$. 10. $14\frac{7}{18}$. 11. $24\frac{1}{18}$. 12. $10\frac{5}{6}$.

Exercise LX.—Page 101.

1. $18\frac{5}{28}$. 2. $16\frac{9}{16}$ gallons. 3. $\frac{43}{90}$.
4. $20\frac{1}{16}$ yards. 5. \$3$\frac{1}{8}$. 6. $101\frac{1}{16}$ acres.
7. $14\frac{3}{20}$ reams. 8. $34\frac{79}{84}$ pounds. 9. $33\frac{1}{476}$ miles.
10. \$38$\frac{1}{15}$. 11. \$98$\frac{23}{35}$. 12. $\frac{14}{63}$, $\frac{33}{70}$, $\frac{1757}{2520}$.

Exercise LXI.—Page 101.

1. \$383$\frac{7}{12}$. 2. \$1$\frac{33}{40}$. 3. 49$\frac{7}{16}$ pounds.
4. \$191$\frac{1}{12}$. 5. 106$\frac{3}{4}\frac{1}{8}$ gallons. 6. 8$\frac{3}{35}$.
7. 145$\frac{1}{7}$ yards; \$403$\frac{3}{8}$. 8. 44$\frac{9}{16}$ pounds. 9. \$5.
10. 10$\frac{3}{84}$ gallons. 11. \$177$\frac{7}{36}$. 12. 774$\frac{1}{63}$ acres.

Exercise LXII.—Page 103.

1. $7\frac{1}{2}$. 2. $6\frac{7}{8}$. 3. $2\frac{1}{4}$. 4. $9\frac{1}{3}$.
5. $37\frac{1}{3}$. 6. $71\frac{1}{2}$. 7. $26\frac{1}{4}$. 8. $10\frac{3}{10}$.
9. $4\frac{4}{7}$. 10. \$30. 11. \$13.14. 12. \$201.
13. \$1.77. 14. \$8.75.

Exercise LXIII.—Page 105.

1. $\frac{3}{17}$. 2. $\frac{1}{8}$. 3. $\frac{7}{64}$. 4. $\frac{7}{136}$. 5. $1\frac{1}{8}$.
6. $\frac{2}{3}$. 7. $\frac{3}{7}$. 8. $8\frac{1}{10}$. 9. $23\frac{31}{2}$. 10. $3\frac{1}{4}$ acres.
11. $2\frac{1}{4}$ cords. 12. $3\frac{5}{6}$ miles. 13. \$$8\frac{2}{3}$.

Exercise LXIV.—Page 107.

1. 15. 2. 40. 3. $35\frac{5}{6}$. 4. $54\frac{1}{4}$. 5. $\frac{18}{25}$.
6. $\frac{3}{50}$. 7. $\frac{3}{5}$. 8. $\frac{7}{44}$. 9. 1. 10. $\frac{10}{63}$.
11. $\frac{7}{18}$. 12. $\frac{20}{63}$. 13. \$$\frac{91}{256}$. 14. \$$\frac{5}{8}$.
15. *A*, \$$281\frac{1}{4}$; *B*, \$225; *C*, \$$303\frac{3}{4}$; \$810.

Exercise LXV.—Page 107.

1. $17\frac{5}{28}$. 2. $49\frac{1}{3}$. 3. 290. 4. 1320. 5. 8789.
6. $61\frac{1}{2}$. 8. $42\frac{3}{4}$ cents. 9. 10 acres. 10. \$$351\frac{9}{176}$. 11. \$$67\frac{3}{8}$.
12. \$$1667\frac{1}{4}$. 13. \$$227\frac{211}{240}$.

Exercise LXVI.—Page 109.

1. 14. 2. 21. 3. $34\frac{2}{7}$. 4. $11\frac{3}{7}$. 5. 3. 6. $\frac{55}{61}$.
7. $1\frac{97}{98}$. 8. $\frac{37}{39}$. 9. $10\frac{5}{6}$. 10. $\frac{8650}{1504}$. 11. $1\frac{18}{28}$. 12. $17\frac{1}{10}$.

Exercise LXVII.—Page 109.

2. 24 bushels. 3. 14 tons. 4. 27 bushels.
5. $24\frac{40}{63}$ weeks. 6. 11 persons. 7. \$$\frac{17}{20}$.
8. $2\frac{4}{7}$ weeks. 9. $11\frac{1}{4}$ bushels. 10. $190\frac{2}{5}$ days.
11. \$$67\frac{1}{2}$. 12. $10\frac{123}{23}$.

Exercise LXVIII.—Page 111.

1. 16. 2. $\frac{3}{4}$. 3. $\frac{5}{27}$. 4. $\frac{3}{26}$. 5. $1\frac{7}{9}$.
6. $4\frac{4}{5}$. 7. 2. 8. $1\frac{5}{9}$. 9. 2. 10. $2\frac{3}{4}$.
11. $7\frac{1}{2}$. 12. $3\frac{4}{7}$. 13. $\frac{4065}{32}$. 14. $1\frac{7303}{2}$. 15. $\frac{5}{11}$.
16. $2\frac{2}{5}$. 17. $38\frac{2}{11}$. 18. 16. 19. $1\frac{54}{55}$. 20. $\frac{17}{45}$.

Exercise LXIX.—Page 114.

1. $\frac{1}{8}$. 2. $3\frac{18}{37}$. 3. 1. 4. $22\frac{2}{1776}$. 5. $3\frac{17}{140}$.
6. $14\frac{23}{35}$. 7. 4. 8. 1. 9. $\frac{11}{30}$. 10. $1\frac{3}{4}$.
11. $\frac{5}{12}$. 12. 15. 13. $\frac{131}{196}$. 14. 1. 15. $\frac{2303}{210}$.
16. $3\frac{41}{2170}$. 17. $\frac{17}{1078}$. 18. $\frac{5}{48}$. 19. $23\frac{5}{21}$. 20. $\frac{23}{35}$.

Examination Papers.—Page 115.

II.

3. $\frac{8}{11}$. 4. $1\frac{7}{8}$; $\frac{11}{36}$. 5. 24 days.

III.

2. $\frac{1}{3}$. 3. 18 bags. 4. \$21900. 5. \$15.85.

IV.

2. $\frac{2}{3}$ of $\frac{9}{16}$, $\frac{2}{5}$, $\frac{3}{7}$, $\frac{7}{15}$, $\frac{13}{25}$. 3. \$5.60. 4. Too large by $\frac{15}{16}$. 5. \$33075.

V.

2. \$359.45. 3. \$660.80. 4. $\frac{2}{3}$. 5. 9 acres.

EXERCISE LXX.—Page 119.

1. $\frac{7}{10}$.	2. $\frac{36}{100}$.	3. $\frac{8}{100}$.	4. $\frac{784}{10000}$.
5. $\frac{709}{10000}$.	6. $\frac{4123}{100000}$.	7. $\frac{814}{10000}$.	8. $\frac{78}{10000}$.
9. $\frac{7614}{100000}$.	10. $\frac{3005}{100000}$.	11. $\frac{427}{100000}$.	12. $\frac{36}{100000}$.
13. $\frac{2007}{10000000}$.	14. $\frac{7124655}{10000000}$.	15. $\frac{6}{1000000}$.	16. ·8.
17. ·71.	18. ·27.	19. ·07.	20. ·136.
21. 2·07.	22. 4·16.	23. 16·126.	24. 126·367.
25. ·18496.	26. 3·00007.	27. 16·00163.	

EXERCISE LXXI.—Page 119.

1. Nine-tenths. 2. Twenty-seven hundredths. 3. Three hundred and sixty-eight thousandths. 4. Sixty-four thousandths. 5. Four, and thirty-one hundredths. 6. Seven, and two hundred and sixteen thousandths. 7. Three, and three hundred and fourteen thousandths. 8. Five, and eight thousand one hundred and sixty-seven ten thousandths. 9. Twenty-one, and three thousand six hundred and one ten-thousandths. 10. Seventeen, and sixty-four ten-thousandths. 11. Eighteen, and eighty-one hundred thousandths. 12. Twenty, and one thousand four hundred and fifty-eight hundred thousandths. 13. ·8; 2·07; ·009. 14. 807·091; 3017·0709; 3·001008. 15. 6·0004; 80·0000609; 101·01001.

EXERCISE LXXII.—Page 120.

1. 65·046.	2. 600·7354.	3. 4475·105045.
4. 2·4397464.	5. 101·209.	6. 10·867.
7. 114·1377.	8. 959·0483.	9. 40·52753.
10. 15156·66886.	11. 200·1211.	12. 25·749445.
13. 227·5021.	14. 122·625 yds.	15. 58·1905 acres.

EXERCISE LXXIII.—Page 121.

1. 16·1524.	2. 2·3806.	3. ·43876.
4. ·23296.	5. 1·8316.	6. ·00521.

7. 3·9249. 8. 1·405. 9. 168·098.
10. ·01. 11. ·6322. 12. 8·3416.
13. 2·5527. 14. 15·809. 15. 173·03863.
16. ·2318 inches. 17. 36·002 grains. 18. ·099.
19. ·146. 20. 13·75 yards.

EXERCISE LXXIV.—Page 122.

1. 15·544. 2. 240·37086. 3. ·0273238.
4. 5·4008. 5. 2474·11. 6. ·26928.
7. 9·6142. 8. ·000072. 9. ·310104.
10. 803·2104. 11. ·040527. 12. 1·010009.
13. 334141·402 sq. in. 14. 9·75 pounds. 15. 334·00692 pounds.
16. 117·04936022 mi. 17. 728·9271. 18. 312·275 pounds.

EXERCISE LXXV.—Page 124.

1. 3·07. 2. 50·615625. 3. 800. 4. ·006446875.
5. 1240. 6. ·00075. 7. ·00016125. 8. ·568.
9. 20200. 10. 22600. 11. ·082. 12. 83.

EXERCISE LXXVI.—Page 125.

1. ·1875. 2. ·75. 3. ·625. 4. ·225.
5. ·15625. 6. ·025. 7. ·0375. 8. ·875.
9. ·06875. 10. ·078125. 11. ·008. 12. 6·6.
13. 24·008. 14. 3·525. 15. 46·3125.

EXERCISE LXXVII.—Page 127.

1. $\frac{1}{3}$. 2. $\frac{6}{11}$. 3. $\frac{27}{37}$. 4. $\frac{329}{999}$. 5. $\frac{4}{165}$. 6. $\frac{283}{900}$.
7. $\frac{1}{148}$. 8. $\frac{1333}{30000}$. 9. $4\frac{79}{1110}$. 10. $11\frac{19}{66}$. 11. $3\frac{118}{999}$. 12. $2\frac{19}{55}$.

EXERCISE LXXVIII.—Page 128.

1. $62{\cdot}\dot{9}20413349443052\dot{}$. 2. $\cdot\dot{2}\dot{4}$; $\cdot 0327118\dot{0}1\dot{5}$.
3. $9{\cdot}92\dot{8}$; $2{\cdot}\dot{2}9\dot{7}$. 4. $3{\cdot}\dot{6}$; $1{\cdot}14\dot{5}$.

EXAMINATION PAPERS.—Page 128.

I.

2. $1\frac{1}{4}$; $\frac{1}{2500}$; ·000011. 3. ·017359; ·0005.
4. ·120508; ·13. 5. $\cdot\dot{7}14285\dot{}$; $\frac{5411}{6600}$.

II.

1. ·375; ·000000375; 356·315375; 160000. 2. ·01825.
4. $\frac{1}{8000}$; $\frac{1}{40}$; $\frac{1}{220}$. 5. 8; 6400.

III.

1. ·9525. 2. 24·975024; 500·5. 3. $\frac{8}{125}$; $15\frac{8}{9}$; 1.
4. $1\frac{3}{4} \times 2\frac{5}{9}$. 5. $\frac{1}{12}$.

IV.

1. 1·1214727. 2. ·54321. 3. \$3000, \$6900.
4. 2520; $3\frac{3}{5}$. 5. $97\frac{9}{14}$.

V.

1. $\frac{2}{3}$. 2. ·0000$\dot{1}$; ·0000999$\dot{9}$. 5. 1·60546875.

EXERCISE LXXIX.—Page 132.

1. 92d. 2. 1104 far. 3. £29 15s. 5d.
4. £309 5s. 5. 2406d. 6. 560d.
7. £1 1s. 5½d. 8. £29 16s. 11d. 9. 183839 far.
10. 3209 far. 11. £328 16s. 4d. 12. 96028 far.

EXERCISE LXXX.—Page 133.

1. 65296 oz. 2. 86 oz. 3. 13 cwt. 52 lb. 13 oz.
4. 2 t. 7 cwt. 74 lb. 1 oz. 5. 4933 oz. 6. 25 t. 16 cwt. 49 lb

EXERCISE LXXXI.—Page 134.

1. 16 oz. 2. 21 lb. 10 oz. 3 dr. 2 scr. 3. 32 lb. 5 dwt.
4. 5460 gr. 5. 1584 gr. 6. 12 lb. 9 oz. 5 dwt. 4 gr.

EXERCISE LXXXII.—Page 135.

1. 65142 in. 2. 1 mi. 66 per. 2 ft.
3. 1 mi. 138 per. 3 yd. 2 ft. 4. 36 ft.
5. 462 ft. 6. 232 fath. 4 ft.

EXERCISE LXXXIII.—Page 136.

1. 12 a. 77 sq. rd. 2. 117900 in. 3. 4 cu. ft. 1557 in.
4. 60 c. 9 cu. ft. 5. 75506904 sq. in. 6. 135424 cu. in.

EXERCISE LXXXIV.—Page 138.

1. 662400 sec. 2. 120 bu. 2 qt. 3. 678 pt.
4. 2311 pt. 5. 335 gal. 2 qt. 1 pt. 6. 556 pk.
7. 61 bu. 25 lb. 8. 1 wk. 2 da. 2 hr. 14 min. 53 sec.
9. 74 bu. 34 lb. 10. 6739740 sec. 11. 12 c. 40 ft.
13. Thursday.

Exercise LXXXV.—Page 140.

1. 174 lb. 3 oz.
2. 74 cwt. 21 lb. 3 oz.
3. 88 rd. 5 yd. 1 ft. 6 in.
4. £34 14s. 8d.
5. 43 bu. 1 pk. 1 pt.
6. 95 rd. 5 yd. 2 ft. 3 in.
7. 6 wk. 3 da. 6 h. 50 min. 33 sec.
8. 22 rd. 2 yd. 8 in.

Exercise LXXXVI.—Page 141.

1. 7 lb. 8 oz. 6 dr. 1 scr. 19 gr.
2. 19 mi. 280 rd. 4 yd. 1 ft. 6 in.
3. 58 ac. 146 sq. rd. 25¼ sq. yd.
4. 5 mi. 311 rd. 5 yd. 2 in.
5. £27 17s. 4d.
6. 158 sq. p. 18 yds. 2 ft. 36 in.
7. 142 bu. 2 pk. 5 qt.
8. 79 lb. 3 oz. 5 dwt. 4 gr.
9. 31 gal. 2 qt. 1 pt.
10. 22 sq. rd. 12 yd. 4 ft. 128 in.
11. 1 cwt. 85 lbs.
12. 72 mi. 253 rd. 1 ft. 6 in.

Exercise LXXXVII.—Page 142.

1. 90 cwt. 82 lbs. 13 oz.
2. 50 lb. 2 oz. 7 dwt. 3 gr.
3. 75 da. 23 h. 34 min. 40 sec.
4. £600 9s. 6¾d.
5. 4985 cwt. 25 lb.
6. 1 lb. 1 oz. 12 dwt.
7. 150 a. 115 sq. rd.
8. 23058 gal.
9. 88 mi. 122 rd. 3 yd.
10. 5 oz. 19 dwt.
11. £280 5s. 9½d.
12. 662 mi. 4 fur. 28 rd. 3 yd. 2 ft. 2 in.
13. 2739 bu. 1 pk. 5 qt.

Exercise LXXXVIII.—Page 143.

1. £15 9s. 7d.
2. 12 lb. 9 oz. 15 dwt. 18 gr.
3. 16 t. 2 cwt. 38 lbs.
4. $\frac{36}{141}$ pt.
5. 2 cu. yd. 6 ft. 960 in.
6. 8.
7. $10\frac{3}{37}$
8. $11\frac{489}{970}$.
9. $5\frac{71}{89}$.
10. 2 bu. 3 pk. 3⅜ qt.
11. 25 demijohns.
12. 5 weeks.

Exercise LXXXIX.—Page 144.

1. 3 pk. 1 qt. 1⅕ pt.
2. 213 rd. 1 yd. 2 ft. 6 in.
3. 4 yd. 2 ft. 5¼ in.
4. 96 rd.
5. 17 cwt. 50 lbs.
6. 88 rd. 26 yd. 8 ft.
7. £1 12s. $10\frac{1}{2}\frac{7}{8}$d.; £5 2s. $8\frac{1}{2}\frac{3}{2}$d.
8. 4 da. 23 hr. 28 min.
9. 1 lb. $7\frac{5}{13}$ oz.

Exercise XC.—Page 145.

1. $\frac{1}{80}$.
2. $\frac{1}{40000}$.
3. $\frac{1}{360}$.
4. $\frac{1}{6563\frac{1}{3}40}$.
5. $\frac{1}{9}$.
6. $\frac{5}{13886}$.
7. $\frac{7}{160000}$.
8. $\frac{713}{30003}$.
9. $\frac{41161}{148875}$.
10. $\frac{3511}{7737}$.
11. $\frac{30}{7}$.
12. $\frac{44}{13}$.

Exercise XCI.—Page 146.

1. 151 sq. rd. 2. 9 oz. 15 dwt. 18 gr. 3. $10\frac{1}{2}$d.
4. 47 min. 6 sec. 5. 11 hr. 55 min. $40\frac{1}{2}$ sec. 6. 8s. 9d.
7. 309 rods. 8. 1556 lb. 4 oz. 9. 12s. $6\frac{3}{4}$d.
10. 3s. $5\frac{1}{2}$d. 11. 2 da. 12 h. 55 min. 21 sec. 12. $\frac{1}{2}$d.

Exercise XCII.—Page 146.

1. £·525. 2. ·282 t. 3. ·78125 oz. 4. ·775 mi.
5. ·3125 pk. 6. £9·26875. 7. 17·895 cwt. 8. 7·875 bu.
9. ·625 fath. 10. ·71. 11. 129·78 hr. 12. ·001625 t.

Exercise XCIII.—Page 147.

1. \$192. 2. \$148.50. 3. \$436.80. 4. \$388.
5. \$2067.50. 6. \$615. 7. \$196.12$\frac{1}{2}$. 8. \$308.
9. \$381.75. 10. \$35.55. 11. \$101.85. 12. \$124.20.
13. \$1668.50. 14. \$44.04. 15. \$32753.12$\frac{1}{2}$.

Exercise XCIV.—Page 148.

2. $12\frac{11}{2}$ lb. 3. \$65.10$\frac{5}{2}$. 4. $3\frac{3}{8}\frac{7}{14}\frac{3}{4}$.
5. $1\frac{8}{4}$. 7. £34 12s. 4d. 8. 5s. 2d.
9. $11\frac{7}{0}\frac{1}{0}$. 10. $6\frac{1}{4}$ oz. 11. \$567.525.
12. \$7152.31$\frac{1}{4}\frac{1}{7}$. 13. \$173.71$\frac{7}{2}$. 14. 7 hr. 11 min. 8 sec.
15. \$2.10. 17. 98 yd. 18. $30\frac{9}{4}$ yd.
19. $4166\frac{2}{3}$ yd. 21. \$1408.80. 22. 3s. $7\frac{8}{17}\frac{6}{3}$d.
23. \$1736.23$\frac{1}{8}$. 24. \$880.

Exercise XCV.—Page 152.

1. 10; 100; 1000; 10; 100. 2. $\frac{1}{10}$; $\frac{1}{100}$; $\frac{1}{1000}$; $\frac{1}{10}$.
4. (1) 172·448 m.; (2) 862·05 m.; ·83 mm.; (3) 365·6 m.; $391\frac{2}{3}\frac{5}{0}$ m.
5. \$272.18$\frac{1}{4}$. 6. \$1801325. 7. 7·6375 ml. 8. 4800 km.

Exercise XCVI.—Page 153.

1. 12000000 sq. m. 2. 835 sq. m. 3. \$155163·648; 101·018 ares.

Exercise XCVII.—Page 154.

1. ·00514 cu. m. 2. 8·765345 cu. m.
3. 1219·2494 0201 cu. m.; 1219249·40201 cu. dm.

Exercise XCVIII.—Page 154.

2. 84 l. 3. 17·54 l. 4. 2 cents. 5. \$10·546875.
6. 1000 l. 7. $4\frac{3}{8}$ gal. 8. \$3.67$\frac{3}{10}$ gained.

Exercise XCIX.—Page 155.

2. .7018 grm. 3. 2·47 grm. 4. 12·021 grm.
5. $72. 6. 30·864 gr. 7. $2{\cdot}15221\frac{3}{7}$ lb.
8. $4479\frac{3}{643}$ grm.; $5443\frac{15}{643}$ grm. 9. 321·76 gr.
10. 1·03575 kgrm.

Exercise C.—Page 157.

1. 72. 2. $15. 3. 23 sheep. 4. $10.50
5. 14 men. 6. 45. 7. 40 %. 8. 75 %.
9. 60 %; $36\frac{4}{11}$ %. 10. 1100. 11. $1200. 12. 57 a.
13. $208\frac{1}{3}$ a. 14. $4. 15. $100.

Exercise CI.—Page 158.

1. $14.40. 2. $15.80. 3. $10. 4. $30.
5. $247. 6. $112. 7. $2500. 8. 7500 bu.
9. $788.75. 10. $1.75. 11. $3\frac{1}{4}$ %. 12. $16.45.
13. 2 %.

Exercise CII.—Page 159.

1. $18. 2. $11.20. 3. $15.20. 4. $11.25.
5. $100. 6. $110.40. 7. $166.25. 8. $65.20.
9. $2500. 10. $7000. 11. $70000. 12. $9600.
13. $5276. 14. $3739.20.

Taxes.—Page 160.

1. $100. 2. $120. 4. $\frac{8}{25}$ ct.; $5.76.
6. $8400. 7. $3000. 8. $15000.

Profit and Loss —Page 161.

1. $1183.50. 2. $567; $1·134. 3. 25 %. 4. 20 %.
5. $33\frac{1}{3}$ %. 6. $1.50. 7. $7225. 8. $135.70
9. $45.80. 10. $1.45. 11. $5.07. 12. $21\frac{1}{2}$ %.

Exercise CIII.—Page 163.

2. $301.53. 3. $12.75. 4. 25 %. 5. $289.80; $19\frac{1}{2}$ %.
7. $400. 8. $500.

Exercise CIV.—Page 164.

1. $30. 2. $38.50. 3. $236·412. 4. $568.05.
5. $451.50. 6. $236.64. 7. $314.60. 8. $45.
9. $729.90\frac{1}{8}$. 10. $198.66. 11. $311.64. 12. $96·738.

13. \$96.11¼. 14. \$882. 15. \$1887.60. 16. \$934.92.
17. \$14·958... 18. \$215.75... 19. \$1306.84. 20. 8 %.
21. 7½ %. 22. 6 %. 23. \$3500. 24. \$32500.
25. \$9000. 26. 3 yrs. 27. 3 yrs. 28. Oct. 4, 1893.
29. 14⅔ yrs.

Exercise CV.—Page 168.

1. \$124.16... 2. \$1215.50... 3. \$6·8992.
4. \$466.56. 5. \$129·856. 6. \$541.21....

Exercise CVI.—Page 169.

1. \$1168.70. 2. \$375. 3. \$900. 4. On 6 mos.
5. \$2100. 6. Gain \$50. 7. \$155.82. 8. \$242.32.
9. \$1.25. 10. The first is cheaper by \$508.76.

Exercise CVII.—Page 171.

1. \$17.20; \$566.80. 2. \$3.60. 3. \$10.35.
4. \$74.53... 5. \$714.28. 6. \$523.80.
7. \$1723.75. 8. \$184. 9. \$336.

Exercise CVIII.—Page 173.

1. 684, 576. 2. \$125, \$225, \$150. 3. \$150, \$200, \$250
4. 112 a.; 98 a. 5. 1520 bu.; 2280 bu. 6. \$6081$0\frac{3}{7}$; \$2918$9\frac{3}{7}$.

Exercise CIX.—Page 174.

1. \$60; \$144; \$96. 2. \$3900; \$1800; \$3300.
3. \$14.40; \$9.60. 4. \$6.30; \$5.25; \$4.20.
5. *A*, \$7507.50; *B*, \$8872.50. 6. \$1080; \$1260; \$1440; \$1620.

Exercise CX.—Page 175.

1. \$21.60; \$23.04; \$25.92. 2. \$525; \$500; \$480.
3. \$1440; \$2385. 4. \$3960; \$2925; \$975.

Exercise CXI.—Page 176.

1. 34·25. 2. 32·22... 3. 493·33... 4. 54·4.
5. 44. 6. 43°.99. 8. \$2.06¼. 9. Lost 30 cts.
10. 44⅛ cts. 11. 12 carats.
12. 225 bbl. at \$6.50; 250 bbl. at \$6.75.
13. \$2. 14. 4 ft. 7 in.

Exercise CXII.—Page 179.

1. 17.	2. 19.	3. 24.	4. 25.
5. 36.	6. 75.	7. 95.	8. 49.
9. 64.	10. 37.	11. 47.	12. 56.
13. 625.	14. 512.	15. 343.	16. 707.
17. 906.	18. 857.	19. 2401.	20. 6325.
21. 5008.	22. 4385.	23. 6671.	24. 7854.
25. ·47.	26. ·27.	27. ·32.	28. ·02.
29. ·0707...	30. ·8449...	31. ·946...	32. ·948...
33. 4·1109...	34. 1·9.	35. 8·426...	36. 2·529...
37. 3·794...	38. 7·08.	39. 73·6.	

Exercise CXIII.—Page 181.

1. 40 rods.	2. $640.	3. 30 ft. square.
4. 125.	5. 66 yd. by 22 yd.	6. 63 ft.; 42 ft.
7. 85 yd.; 85 cts.	8. 246 ft. 11·39 in.	9. 250 ft.; 125 ft.
10. 57.	11. 20 yd. by 4 yd.	12. $1496.

Exercise CXIV.—Page 183.

1. 96 sq. ft.	2. 91 sq. ft.	3. 525 sq. ft.	4. $18\frac{2}{3}$ sq. yd.
5. $351\frac{1}{3}$ sq. yd.	6. 470 sq. ft.	7. $\$14.43\frac{1}{9}$.	8. $4\frac{1}{8}$ acres.
9. 1404 bricks.	10. 5 sq. ft. $142\frac{1}{2}$ sq. in.		

Exercise CXV.—Page 183.

1. 84 yd.	2. $26\frac{2}{3}$ yd.	3. 64 yd.	4. $138\frac{2}{3}$ yd.
5. $48.	6. $57.60.	7. $\$29.16\frac{2}{3}$.	

Exercise CXVI.—Page 184.

1. 126 yd. 2. 578 sq. ft. 3. $115\frac{1}{3}$ yd. 4. $57\frac{3}{5}$ yd. 5. $6.80.

Exercise CXVII.—Page 185.

1. $114\frac{6}{11}$ sq. rods. 2. $91.94... 3. 6. 4. 192 sq. ft.

Exercise CXVIII.—Page 186.

1. 144 sq. ft.	2. 4 times as great.	3. $26.88.
4. $32·256.	5. 4 chains.	

Exercise CXIX.—Page 188.

1. 22·02.. ft. 2. 19·2... ft. 3. 30 ft. 4. 96 ft. 5. 85 ft.

Exercise CXX.—Page 189.

1. $176\frac{1}{4}\frac{1}{4}$ yd. **2.** $50\frac{1}{1}\frac{9}{9}$ acres. **3.** $150\frac{9}{9}$ ft. **4.** $412\frac{1}{2}$ ft.
5. ·4543... ml.

Exercise CXXI.—Page 190.

1. 240 cu. ft. **2.** 95 cu. ft. **3.** $187\frac{1}{2}$ cu. ft.
4. $45\frac{1}{2}$ cu. ft. **5.** 16000. **6.** $\$116\frac{4}{7}$.
7. 9 ft. **8.** $4\frac{4}{5}$ ft. **9.** 21·97 cords.
10. 287280 cu. in.; 1036·087 gal. **11.** 2·769 gal.
12. $562\frac{1}{2}$ lb. **13.** $\frac{5}{2\,6\,0}$ in. **14.** $16\frac{2}{3}$ ft.
15. 22·005... in. **16.** 28·8827 in.

Miscellaneous Problems.—Page 192.

2. 16 days. **3.** 5 days. **4.** 9 mos. **5.** 5 days.
7. $6\frac{2}{3}$ hr. **8.** 8 days. **9.** $3\frac{7}{1}$ days. **10.** $13\frac{1}{2}$ days.
11. $14\frac{2}{7}$ days. **12.** $13\frac{1}{7}$ days. **14.** $5\frac{1}{2}\frac{1}{7}$ days. **15.** 15 days.
16. 24 days. **17.** $8\frac{2}{1}$ hrs. **19.** $3\frac{2}{7}\frac{5}{5}$ days. **20.** $13\frac{1}{1}\frac{1}{3}$ days.
22. Man $27\frac{1}{3}$ da.; boy 120 da. **23.** Man 90 da.; boy 180 da.
24. Man 28 da.; woman 40 da. **25.** Woman 30 da.; boy 40 da.
27. $24\frac{1}{7}$ min. **28.** 24 hrs. **29.** $23\frac{1}{4}$ min. **30.** $17\frac{1}{7}$ min.
32. 180 min. **33.** 72 min. **35.** \$23030. **36.** 35 ft.
37. \$1582. **38.** \$2843.75. **40.** \$10500. **42.** 8 days.
43. \$1827. **44.** $38\frac{4}{7}$ days. **45.** \$94.50.
48. 7 h, $38\frac{2}{1}\frac{2}{7}$ min. **49.** 3 h. $16\frac{4}{1}\frac{4}{7}$ min.
51. 6 h. 24 min.; 6 h. $41\frac{5}{1}\frac{5}{7}$ min.
52. 4 h. $18\frac{6}{1}\frac{6}{3}$ min., 4 h. $21\frac{0}{1}\frac{0}{1}$ min. **53.** 4 h. $54\frac{6}{1}\frac{6}{1}$ min.; 6 h.
54. 1 h. $4\frac{8}{1}\frac{8}{2}$ min., 3 h. $13\frac{1}{1}\frac{1}{4}$ min., 6 h. $27\frac{8}{1}\frac{8}{3}$ min., 8 h. $36\frac{1}{1}\frac{1}{3}$ min
55. 4 h. $5\frac{2}{1}\frac{2}{1}$ min., 4 h. $38\frac{2}{1}\frac{2}{1}$ min. **56.** 1 h. $38\frac{2}{1}\frac{2}{1}$ min.
59. $7\frac{3}{4}$ ml. **60.** 3 hr. **62.** $\frac{1}{3\,2}$ distance.
63. 45 min., 135 min. **64.** 5 ml., 1 ml. **66.** 10 oranges.
67. 65 glasses. **69.** \$10. **70.** \$11520.
72. 48 mi. **73.** 24 ml. **75.** 180 leaps.
76. Man 190 steps; boy 285 steps.

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SHORT CLEAR HINTS AND SUGGESTIONS

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There is no other book that can rival the Public School Algebra as an introductory text-book. *It has been prepared with a view to meet precisely the contemplated standard for* ENTRANCE EXAMINATIONS IN ONTARIO AFTER 1900. It will be found right up to the requirements of the 20TH CENTURY.

It is **Condensed, Original, Helpful** and will win its way wherever it is tried

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